Research summary related to interacting particle models:

- Interacting particle model for fish migration
  - Behavior of the particle model with noise (with B. Birnir, K. Taylor; simulation assistance from P. Trethewey, L. Youseff)
  - Parallelization of the simulations (with B. Birnir, J. Gilbert, P. Trethewey, L. Youseff)
  - The model applied to the Icelandic Capelin (with B. Birnir, B. Einarsson, S. Sigurdsson, The Marine Institute of Iceland)
  - Scaling in interacting particle systems (with B. Einarsson) [current]

- Interacting particle models for gang dynamics
  - A coupled network model for gang rivalry formation (with R. Hegemann, L. Smith, S. Reid, A. Bertozzi, G. Tita)
  - A statistical mechanics approach to gang territorial development (with L. Chayes, M. R. D’Orsogna)

- Kinetic and hydrodynamic models for particle systems
  - Phase transition and diffusion among socially interacting self-propelled agents (with P. Degond)
  - Phase transition in a kinetic Cucker-Smale model with self-propulsion and friction (with J. A. Carrillo, P. Degond) [current]
  - A kinetic contagion model for fear in crowds (with J. Rosado) [current]
  - An exploration of the effect of normalization and different kinds of noise in Vicsek-type flocking models (with M. Burger) [current]
The Data: Icelandic stock of capelin
The Icelandic stock of capelin
An example of the acoustic data:
Our model

\[
\left( \begin{array}{c} x_k(t + \Delta t) \\ y_k(t + \Delta t) \end{array} \right) = \left( \begin{array}{c} x_k(t) \\ y_k(t) \end{array} \right) + \Delta t \cdot v_k(t) \frac{D_k(t)}{\|D_k(t)\|} + C(\tilde{p}_k(t))
\]

- Here, \(D_k\) is the directional heading of particle \(k\)
- \(\Delta t\) is the timestep
- Particle \(k\)'s position in the plane is \(p_k\)
- \(\tilde{p}_k\) is the nearest gridpoint to \(p_k\)
- \(C(\tilde{p}_k)\) is the current at \(\tilde{p}_k\)
Choosing the direction

The directional heading of the particles (apart from environmental effects) is determined as follows:

\[
\begin{pmatrix}
\cos(\phi_k(t + \Delta t)) \\
\sin(\phi_k(t + \Delta t))
\end{pmatrix} = \frac{\mathbf{d}_k(t + \Delta t)}{\| \mathbf{d}_k(t + \Delta t) \|}
\]

where

\[
\mathbf{d}_k(t + \Delta t) := \left( \sum_{r \in R_k} \frac{\mathbf{p}_k(t) - \mathbf{p}_r(t)}{\| \mathbf{p}_k(t) - \mathbf{p}_r(t) \|} + \sum_{o \in O_k} \left( \cos(\phi_o(t)) \right) + \sum_{a \in A_k} \frac{\mathbf{p}_a(t) - \mathbf{p}_k(t)}{\| \mathbf{p}_a(t) - \mathbf{p}_k(t) \|} \right).
\]
February, 1985  
February 1991  
February 2008

1 http://www.wetterzentrale.de/topkarten/fsfaxsem.html
Function $r(T)$ determines the reaction of the particles to the temperature field.

$$r(T) := \begin{cases} 
-(T - T_1)^4 & \text{if } T \leq T_1 \\
0 & \text{if } T_1 \leq T \leq T_2 \\
-(T - T_2)^2 & \text{if } T_2 \leq T 
\end{cases}$$
The model

\[
\begin{pmatrix}
  x_k(t + \Delta t) \\
  y_k(t + \Delta t)
\end{pmatrix}
= \begin{pmatrix}
  x_k(t) \\
  y_k(t)
\end{pmatrix} + \Delta t \cdot v_k(t) \frac{D_k(t)}{\|D_k(t)\|} + C(\ddot{p}_k(t))
\]

where

\[
D_k(t + \Delta t) := \alpha \begin{pmatrix}
  \cos(\phi_k(t + \Delta t)) \\
  \sin(\phi_k(t + \Delta t))
\end{pmatrix} + \beta \frac{\nabla r(T(p_k(t)))}{\|\nabla r(T(p_k(t)))\|}
\]

for \(\alpha + \beta = 1\).
Figure: The distribution of capelin during the spawning migration of 1984-1985.
(a) Acoustic data from November 1 to November 21  (b) Acoustic data from January 14 to February 8
(c) Close up of the distribution of capelin from February 7 to February 20 of 1985.
Figure: The distribution of capelin during the spawning migration of 1991.
(a) Acoustic data from January 4 to January 11.
(b) Close up of the distribution of capelin southeast of Iceland from February 8 to February 9 of 1991.
(c) Acoustic data from February 17 to February 18.
Figure: Simulation of the 2007-2008 spawning migration.
(a) Early January, day 0
(b) Mid-February, day 47
(c) Late February, day 59
(d) Early March, day 65.
Figure: Collected migration data:
(a) Measured distribution of capelin near south coast of Iceland from February 26 to February 27 of 2008.
(b) Measured distribution of capelin near the southeast coast of Iceland from February 29 to March 3 of 2008.
We measure the sensitivity of the system by seeing how the migration route and timing change. For details, see to [2].
In the real migrations which we are trying to accurately capture, it is safe to assume there are around $5 \cdot 10^{10}$ fish.
The problem of superindivduals

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- This means each particle represents $10^6$ fish.
- Each particle must therefore be thought of as a superindividual.
The problem of superindividuals

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- In our simulations, we use roughly $5 \cdot 10^4$ particles.
- This means each particle represents $10^6$ fish.
- Each particle must therefore be thought of as a superindividual.
- With these superindividuals, we captured the migration.
The goal

- One fish per particle
- Then we could more confidently justify our behavioral rules, since they are based on data obtained from interactions among individual fish

So this leads to a question: how does the system change as we change the number of particles?
The goal

- One fish per particle
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So this leads to a question: how does the system change as we change the number of particles?

We need to make some assumptions:

- We assume uniform density of particles and fish in the schools
- The interaction length of the particles should be much less than the size of the school
- We further assume the velocities of the particles are equal
If sufficiently dense, local interactions between particles allows information to propagate through a school
- Temperature information
- Information about predators
- Information about food

We want to preserve the speed at which this information propagates through the school.
Varying Numbers of Particles

[Diagrams of varying particle densities]
Varying Numbers of Particles
Varying Numbers of Particles

A. Barbaro (CWRU)

Modeling fish migration with an interacting particle model

17 April 2015 21 / 43
Varying Numbers of Particles
In the actual migration, there are a given number of fish within a given area. Each simulation needs to relate back to this real situation:
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Each simulation needs to relate back to this real situation:

\[
\frac{\text{# Real fish}}{\text{Size of domain}} = \left( \frac{\text{# Real fish}}{\text{# Particles in simulation}} \right) \left( \frac{\text{# Particles in simulation}}{\text{# Zones of interaction}} \right) \left( \frac{\text{# Zones of interaction}}{\text{Size of domain}} \right)
\]

\[
= \left( \frac{\text{Fish per particle}}{\text{# Particles in simulation}} \right) \left( \frac{\text{# Particles in simulation}}{\text{# Zones of interaction}} \right) \left( \frac{\text{# Zones of interaction}}{\text{Size of domain}} \right)
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\[
= \left( \frac{F}{N_i} \right) \left( \frac{N_i}{D/\pi R_i^2} \right) \left( \frac{D/\pi R_i^2}{D} \right)
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When particles are uniformly distributed, the second term is roughly the number of interaction neighbors per particle, which is close to uniform in space. Calling this \(M_i\) gives:
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Index the simulation where we captured the migration by 0.

Index a new simulation by 1.

# Real fish
Size of domain remains constant, so:

\[
\frac{N_0}{M_0} (\frac{1}{\pi R_0^2}) = \frac{N_1}{M_1} (\frac{1}{\pi R_1^2})
\]

\[
\Rightarrow \frac{1}{N_0} M_0 (\frac{1}{R_0}) = \frac{1}{N_1} M_1 (\frac{1}{R_1})
\]

Consider number of interaction partners to be fixed. Then:

\[
R_1 = R_0 \sqrt{\frac{N_1}{N_0}}
\]

So, if we want to maintain the number of interaction partners, the radii and the number of particles should relate as follows:

\[
R \propto \frac{1}{\sqrt{N}}
\]
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\[ R_1^2 = \frac{R_0^2 N_0}{N_1} \Rightarrow R_1 = R_0 \sqrt{\frac{N_0}{N_1}} \frac{1}{\sqrt{N_1}} \]
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To guarantee this, \( v\Delta t = cR \) where \( v \) and \( c \) are constant as we vary the number of particles
In discrete system, want the portion of the zone traversed per timestep to remain constant as we vary the number of particles

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To guarantee this, $\nu \Delta t = cR$ where $\nu$ and $c$ are constant as we vary the number of particles

In this way, we see:

$$\Delta t \propto R \propto \frac{1}{\sqrt{N}}.$$
In the actual migration, there are a given number of fish within a given area.

Each simulation needs to relate back to this real situation.

Schematic:

\[
\frac{\text{fish}}{\text{region}} = \left( \frac{\text{particles}}{\text{interaction-zone}} \right) \left( \frac{\text{fish}}{\text{particle}} \right) \left( \frac{\text{interaction-zone}}{\text{region}} \right)
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Let \( N \) denote the total number of particles in a simulation, \( F \) denote the number of fish in the migration, and \( A_w \) denote the total area of the region
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Let $N$ denote the total number of particles in a simulation, $F$ denote the number of fish in the migration, and $A_w$ denote the total area of the region.

Let $M$ denote the number of particles per interaction zone.

- Constant across interaction zones due to constant density assumption.
- For computational intensity, need $M$ is constant across different simulations (so the number of neighbors for each particle remains constant).
Then for a given simulation indexed by $i$, \[ \frac{F}{A_w} = (M)(\frac{F}{N_i})(\frac{A_w}{\pi r_i^2}) \Rightarrow \frac{1}{A_w^2} = \frac{M}{(\pi r_i^2)N_i} \]
Relating time and space to the number of particles

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For two different simulations:

\[
\frac{M}{(\pi r_0^2)N_0} = \frac{M}{(\pi r_1^2)N_1} \Rightarrow \left( \frac{r_1}{r_0} \right)^2 = \frac{N_0}{N_1}
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Relating time and space to the number of particles

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- $r_1 = r_0 \sqrt{\frac{N_0}{N_1}}$

Considering $r_0$ and $N_0$ to have come from a reference simulation:

- $\Delta t \propto r \propto \sqrt{\frac{1}{N}}$
Our parameters for the migrations

- $\Delta t = 0.05$ days
- Initial speed $v_k \approx 4 - 8$ km/day
- $r_r = 0.01$ or about $\sim 120$ m
- $r_o = r_a = 0.1$ or about $\sim 1.2$ km
- Number of particles is roughly $5 \cdot 10^4$
Scaling down to an individual level

How do the particles scale as we take $N^s$ to 1? A rough estimate for the total number of fish in a migration is $F \simeq 5 \cdot 10^{10}$. 

Radii scale with $\Delta q$, so $r_0 \simeq 120$ meters $\Rightarrow r_1 \simeq 12$cm, 

and $\Delta q_0 \simeq 1.2$ km $\Rightarrow \Delta q_1 \simeq 1.2$ meters. 

These are all biologically reasonable!
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- $\Delta t_0 = 0.05$ days and $\frac{\Delta t_0}{\Delta q_0} = \frac{\Delta t_1}{\Delta q_1} \Rightarrow \Delta t_1 = 4.32$ seconds
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Radii scale with $\Delta q$, so

- $r_{0} \approx 120 \text{ meters} \Rightarrow r_{1} \approx 12 \text{ cm}$
- $r_{00} = r_{a0} \approx 1.2 \text{ km} \Rightarrow r_{01} = r_{a1} \approx 1.2 \text{ m}$
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These are all biologically reasonable!
Toward Data

- Einarsson, Birnir, and Sigurdsson have created a dynamic energy budget (DEB) model for the physiology of the capelin [9]
- Next step: Incorporate this DEB model into the simulations of the spawning migration

Toward Mathematics
Where to go from here

Toward Data
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- Next step: Incorporate this DEB model into the simulations of the spawning migration

Toward Mathematics
- Numerical validation of the proposed scaling laws
- Kinetic and hydrodynamic versions of similar models have been and are being studied
- Models taking into consideration the number of interaction neighbors have also been proposed and studied
- Including emotional influences into the model
Gangs are a problem in Los Angeles!
29 Active Gangs in Hollenbeck
69 Rivalries Among the Gangs
A Set Space is a gang's center of activity where gang members spend a large quantity of their time
Gang activity in Hollenbeck is generally isolated from gang activity outside of Hollenbeck
Freeways and other geographic features influence the rivalry network

---

A control: just diffusion
What’s going wrong?
Potential models

- Graph Generating Methods
  - Geographical Threshold Graphs
Potential models

- Graph Generating Methods
  - Geographical Threshold Graphs

- Agent-Based Methods
  - Brownian Motion with Semi-Permeable Boundaries
  - Biased Lévy Flights with Semi-Permeable Boundaries
Potential models

- **Graph Generating Methods**
  - Geographical Threshold Graphs

- **Agent-Based Methods**
  - Brownian Motion with Semi-Permeable Boundaries
  - Biased Lévy Flights with Semi-Permeable Boundaries
    - Coupling the rivalry network and avoidance strength
    - Decay on the edges of graph
    - Heading home
    - Avoiding rivals’ set spaces
    - Semi-permeable freeways
Geographical Threshold Graphs (GTGs) randomly assign weights $\eta_i$ to the $N$ nodes.

The edge between nodes $n_i$ and $n_j$ exists only if $\frac{F(\eta_i, \eta_j)}{d(n_i, n_j)^\beta} \geq \text{Threshold}$

We construct a specific realization of GTGs:

- $\eta_i = \text{size of gang } i$
- $F(\eta_i, \eta_j) = \eta_i \cdot \eta_j$, and $\beta = 2$
- Threshold to have the same number of rivalries as observed network

---

Movement dynamics:

- Agents move in free space according to a biased Lévy walk
- Choose direction of bias according to location of other gangs’ set spaces and location of the agent’s own set space
- Agents have some probability of crossing a boundary

Interactions:

- If two gang members from different gangs cross paths, then an interaction has occurred and the rivalry between the gangs is excited
- At the end of the simulation, we exclude rivalries where the number of interactions is mutually insignificant to both gangs
Density of Ensemble SBLN Networks at Each Iteration

- Red line: Average Density of Simulated Networks
- Green line: True Density
- Blue line: Density of Simulated Networks

Iteration Number

Density of Graph
<table>
<thead>
<tr>
<th></th>
<th>Density</th>
<th>Variance</th>
<th>Centrality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>0.169951</td>
<td>4.32105</td>
<td>0.201058</td>
</tr>
<tr>
<td>GTG</td>
<td>0.169951</td>
<td>9.976219</td>
<td>0.277778</td>
</tr>
<tr>
<td>Ensemble BSN</td>
<td>0.163547 ± 0.005593</td>
<td>3.6642331 ± 0.483954</td>
<td>0.1503968 ± 0.018831</td>
</tr>
</tbody>
</table>

**Table:** The table provides the shape measures for the observed network, GTG, BMN, and ensemble BSN.
Performance of the models

- SBLN and GTG both performed quite well in metric comparisons (accuracy, shape, community structure metrics)
- SBLN allows us to explore *evolution* of the rivalries
- SBLN produces dynamic stochastic networks:

![Graphs showing dynamic stochastic networks](image)

Comparison (left to right) of ensemble SBLN 100% edge agreement, ensemble SBLN 50% edge agreement, and ensemble SBLN 1% edge agreement
SBLN allows us to see where interactions take place.


M. Bostan and J. A. Carrillo, *Asymptotic Fixed-Speed Reduced Dynamics for Kinetic Equations in Swarming*, Preprint UAB


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