predictability
predictability

predictability = knowing initial conditions
predictability

predictability = knowing parametric conditions
the millennium bridge, London
traveling waves in measles epidemic

predictability in global disease dynamics
1760
MATHEMATICS IN EPIDEMIOLOGY

DANIEL BERNOULLI
(1700–1782)

Mem Math Phy Acad Roy Sci Paris 1766

I simply wish that, in a matter which so closely concerns the wellbeing of the human race, no decision shall be made without all the knowledge which a little analysis and calculation can provide.

Daniel Bernoulli 1760.

• model for smallpox inoculation
• use of differential equations
• published in 1766
• the use of calculus, in particular differential equations was novel
inoculation debate

pro

Charles Marie de la Condamine (1701–1774)

con

Pierre Louis Moreau de Maupertuis (1698–1759)
D’ALEMBERT WAS A MEMBER OF THE ROYAL SOCIETY

KNEW OF BERNOULLI’S WORK SINCE 1760 PRESENTATION BY CONDAMINE

GENERALIZED BERNOULLI’S MODEL AND PUBLISHED IT BEFORE HIM IN 1765

D’ALEMBERT DID NOT LIKE LEONARD EULER, EULER WAS A GOOD FRIEND OF BERNOULLI HOWEVER.

BERNOULLI WAS VERY ANGRY ABOUT D’ALEMBERT’S BEHAVIOUR
What do you say about the enormous platitudes of the great d'Alembert about the probabilities; as I find myself too frequently unjustly treated in his publications, I have decided already some time ago to read nothing anymore which comes from his pen; I have taken this decision on the occasion of a manuscript about inoculation which I sent to the Academy in Paris eight years ago and which was greatly appreciated because of the novelty of the analysis; it was, I dare say, like incorporating a new province into the body of mathematics; it seems that the success of this new analysis caused him pains of the heart;
he has criticized it in a thousand ways all equally ridiculous, and after having it well criticized, he pretends to be the first author of a theory which he did not only hear mentioned. He, however, knew that my manuscript could only appear after some seven or eight years, and he could only have knowledge about it in his capacity as member of the Academy, and in this respect my manuscript should have stayed sacred until it was made public. Dolus an virtus quis in hoste requirat!
global mobility and dense populations

• > 50% urban population
• population > 7 billion
• 3 billion passengers / yr
• 5 trillion km / yr
H1N1 (swine flue) 2009

Pandemic (H1N1) 2009,
Number of laboratory confirmed cases as reported to WHO

Status as of 01 July 2009
09:00 GMT

The boundaries and names shown and the designations used on this map do not imply the expression of any opinion whatsoever on the part of the World Health Organization concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. Dotted lines on maps represent approximate border lines for which there may not yet be full agreement.

Map produced: 01 July 2009 11:38 GMT

Data Source: World Health Organization
Map Production: Public Health Information and Geographic Information Systems (GIS) World Health Organization

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EHEC 2011

- foodborne disease
- E. Coli
- May - June 2011
- 4000 affected, 60 died

finding outbreaks

Tracing the Origin and Spread of Agriculture in Europe
Ran Pinhasi**, Jeanne Fox**, Albert J. Ammerman**
1 School of Anatomy and Human Sciences, The Hebrew University, Jerusalem, Israel; 2 School of Archaeology, University of Oxford, United Kingdom; 3 Department of Earth and Environmental Sciences, The Hebrew University, Jerusalem, Israel.

Spatial gradients in Clovis-age radiocarbon dates across North America suggest rapid colonization from the north
Marvin J. Hamilhao**: and Brian J. Buchanan**

Cultural hitchhiking on the wave of advance of beneficial technologies


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questions

- Speed?
- Arrival?
- Origin?

\[ T = \frac{D}{V} \]

SIR DYNAMICS

The basic reproduction number

\[ \alpha \times T = R_0 > 1 \]

mean field dynamics:

\[
\begin{align*}
\partial_t S &= -R_0 IS \\
\partial_t I &= R_0 IS - I \\
R &= 1 - S - I
\end{align*}
\]
reaction diffusion models

\[ \partial_t I = \alpha IS/N - \beta I + D \partial_x^2 I \]
\[ \partial_t S = -\alpha IS/N + D \partial_x^2 S \]

\[ v \propto \sqrt{(R_0 - 1)D} \]
\[
\partial_t j_n = \alpha s_n j_n - \beta j_n + \gamma \sum_{m \neq n} P_{nm} (j_m - j_n).
\]
\[
\partial_t s_n = -\alpha s_n j_n + \gamma \sum_{m \neq n} P_{nm} (s_m - s_n).
\]
geographic distance and arrival times in a hypothetical pandemic
quantitative computational models

GLEaMviz


Colizza et al. The role of the airline transportation network in the prediction and predictability of global epidemics. PNAS (2006)


measuring human mobility
THE FLUX OF DOLLARS
$$w_{ij} \sim w^H_{ij}$$
A MODEL FOR THE SPREAD OF H1N1 IN THE US

Cases in county:
- None
- 1 or more
- 5 or more
- 25 or more
- 100 or more
- 200 or more

Infecteds over time:

Time

0 2 4 6 8 10 12 14 16 18 20

0 50 100 150 200 250 300
H1NI PROJECTIONS US

Cases in county:
- None
- 1 or more
- 5 or more
- 25 or more
- 100 or more
- 200 or more

- Chicago Area: 1100-1500
- NY Area: 1100-1600
- Miami Area: 65-190
- Dallas Area: 65-190
- LA Area: 400-700

May 17th, 2009
projected May 2nd
H1N1 PANDEMIC 2009

4453 cases in the US
Confirmed cases as of May 18
RoCS ESAM Northwestern
http://rocs.northwestern.edu

6,600 to 7,900 cases in the US
Projected as of May 17 of a worst-case scenario starting on 5/6/2009

RoCS ESAM Northwestern
http://rocs.northwestern.edu
QUANTITATIVE MODELS

assume count: 18

Day 28
Day 49
Day 70

http://rocs.northwestern.edu
Northwestern University
Uses and Abuses of Mathematics in Biology

Robert M. May

In the physical sciences, mathematical theory and experimental investigation have always marched together. Mathematics has been less intrusive in the life sciences, possibly because they have until recently been largely descriptive, lacking the invariance principles and fundamental natural constants of physics. Increasingly in recent decades, however, mathematics has become pervasive in biology, taking many different forms: statistics in experimental design; pattern seeking in bioinformatics; models in evolution, ecology, and epidemiology; and much else. I offer an opinionated overview of such uses—and abuses.

Darwin once wrote "I have deeply regretted that I did not proceed far enough at least to understand something of the great leading principles of mathematics, for men thus endowed seem to have an extra sense." With the benefit of hindsight, we can see how much an "extra sense" could indeed have solved one of Darwin’s major problems. In his day, it was thought that inheritance "blended" maternal and paternal characteristics. However, as pointed out to Darwin by the engineer Fleeming Jenkin and others, with blending inheritance it is virtually impossible to preserve the natural variation within populations that is both observed and essential to his theory of how evolution works. Mendel’s observations on the particular nature of inheritance were contemporary with Darwin, and his published work accessible to Darwin. Fisher and others have suggested that Fleeming Jenkin’s fundamental and intractable objections to *The Origin of Species* could have been resolved by Darwin or one of his colleagues, if only they had grasped the mathematical significance of Mendel’s results (1). But half a century elapsed before Hardy and Weinberg (H-W) resolved the difficulties by proving that partiliculate inheritance preserved variation within populations (2).

Today, the H-W Law stands as a kind of Newton’s First Law (bodies remain in their state of rest or uniform motion in a straight line, except as acted upon by external forces) for evolution: Gene frequencies in a
The increasing speed and sophistication and ease of use of computers enables an increasingly large number of life scientists who have no substantial background in mathematics to explore “mathematical models” and draw conclusions about them.
Such activity usually consists of representing sensible and evidence-based assumptions as the starting point for a complicated and usually nonlinear dynamical system, assigning particular parameters (often in an arbitrary way), and then letting this complicated system rip.
we arguably are seeing an increasingly large body of work in which sweeping conclusions - “emergent phenomena” - are drawn from the alleged working of a mathematical model, without clear understanding of what is actually going on. I think this can be worrying.
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Projection for May 17 of a worst-case scenario starting on 5/6/2009

6,600 to 7,900 cases in the US

RoCS ESAM Northwestern

http://rocs.northwestern.edu
is something more fundamental going on?
redefining the notion of distance
simple idea: effective distance

\[ P(i|j) : \text{fraction of traffic: } j \to i \]

- JFK, New York
- LHR, New York
- FRA, Frankfurt
- PIT, Pittsburgh, PA
- RDU, Raleigh Durham, NC
a different notion of distance

\[ d(i|j) = 1 - \log P(i|j) \]
shortest path trees

\[ d(i|j) = 1 - \log P(i|j) \quad \lambda(\Gamma) = L - \sum_{i=1}^{L-1} \log P(i|j) \]

\[ D(m|n) = \min_{\Gamma} \lambda(\Gamma) \]

Guangzhou
Hong Kong International Apt

Tokyo Haneda Apt

Seoul Incheon International Airport

Tokyo Narita Apt

Beijing Capital Apt

Shanghai Pudong International Apt

Shanghai Hongqiao International Apt

Taipei Taiwan Taoyuan International Apt

Mumbai

Bangkok Suvarnabhumi International Apt

Jakarta Soekarno−Hatta Apt

Kuala Lumpur International Airport

Singapore Changi Apt

Amsterdam

Barcelona Apt

Paris Charles de Gaulle Apt

Copenhagen Apt

Rome Fiumicino Apt

Frankfurt International Apt

London Gatwick Apt

London Heathrow Apt

Madrid Barajas Apt

Munich International Airport

Milan Malpensa Apt

Paris Orly Apt

London Stansted Apt

Zurich Airport

Dubai

Atlanta Hartsfield−Jackson Intl Apt

Boston Logan International Apt

Baltimore Washington International Apt

Charlotte

Cincinnati Northern Kentucky Intl Apt

Washington Ronald Reagan National Apt

Denver Intl Apt

Dallas/Fort Worth Intl Apt

Detroit Wayne County

Newark Liberty International Apt

Fort Lauderdale/Hollywood Intl Apt

Washington Dulles International Apt

Houston George Bush Intercontinental Apt

New York J F Kennedy International Apt

Las Vegas McCarran International Apt

Los Angeles International Apt

New York La Guardia Apt

Chicago Midway Apt

Mexico City Juarez International Apt

Miami International Apt

Minneapolis International Apt

Chicago O'Hare International Apt

Philadelphia International Apt

Seattle/Tacoma International Apt

San Francisco International Apt

Salt Lake City

Toronto Lester B Pearson Intl Apt

Melbourne Airport

Sydney Kingsford Smith Apt

Phoenix Sky Harbor Intl Apt.
propagating waves
the spread in effective distance
reconstructing an outbreak origin
outbreak origin reconstruction

IO: Atlanta, $t=63$ days

LAX CRP DFW ORD ATL LHR EIN LBV AWZ MDC HND

$t = 35$ days

$t = 49$ days

$t = 63$ days

$t = 77$ days
outbreak origin reconstruction

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outbreak origin reconstruction

IO: Atlanta, t=63 days

0 5 10 15 20 25
0 5 10 15 20 25
0 5 10 15 20 25
0 5 10 15 20 25

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a key component of predictability

ongoing projects

HUS/EHEC 2011

neolithic spread of farming

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