# Statistical Mechanics of Money, Income, Debt, and Energy Consumption

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- *European Physical Journal B* **17**, 723 (2000) > …………… >
- *Reviews of Modern Physics* **81**, 1703 (2009)
- Book *Classical Econophysics* (Routledge, 2009)
- *Entropy* **15**, 5565 (2013).

**Outline:**

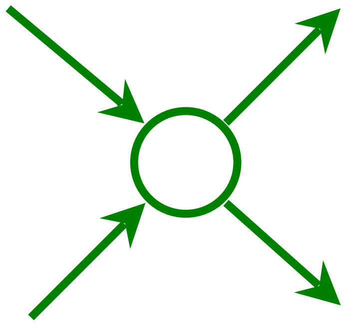
- Statistical mechanics of money
- Debt and financial instability
- Two-class structure of income distribution
- Global inequality in energy consumption

INET funding 2013

# Boltzmann-Gibbs probability distribution of **money**

Collisions between atoms

$\varepsilon_1$



$\varepsilon_1' = \varepsilon_1 + \Delta\varepsilon$

$\varepsilon_2$

$\varepsilon_2' = \varepsilon_2 - \Delta\varepsilon$

Conservation of energy:

$\varepsilon_1 + \varepsilon_2 = \varepsilon_1' + \varepsilon_2'$

Detailed balance:

$\cancel{w_{12\to1'2'}}P(\varepsilon_1)\,P(\varepsilon_2) = \cancel{w_{1'2'\to12}}P(\varepsilon_1')\,P(\varepsilon_2')$

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$ of energy $\varepsilon$, where $T = \langle\varepsilon\rangle$ is temperature. It is **universal** – independent of model rules, provided the model belongs to the time-reversal symmetry class.

Boltzmann-Gibbs distribution maximizes entropy $S = -\Sigma_\varepsilon P(\varepsilon)\ \ln P(\varepsilon)$ under the constraint of conservation law $\Sigma_\varepsilon P(\varepsilon)\ \varepsilon = \text{const}$.

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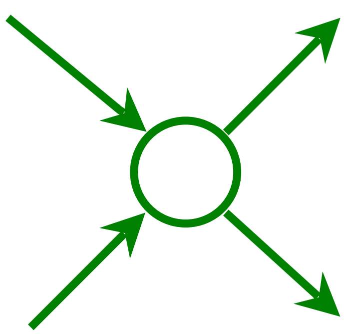
Economic transactions between agents

$m_1$



$m_1' = m_1 + \Delta m$

$m_2$

$m_2' = m_2 - \Delta m$

Conservation of money:

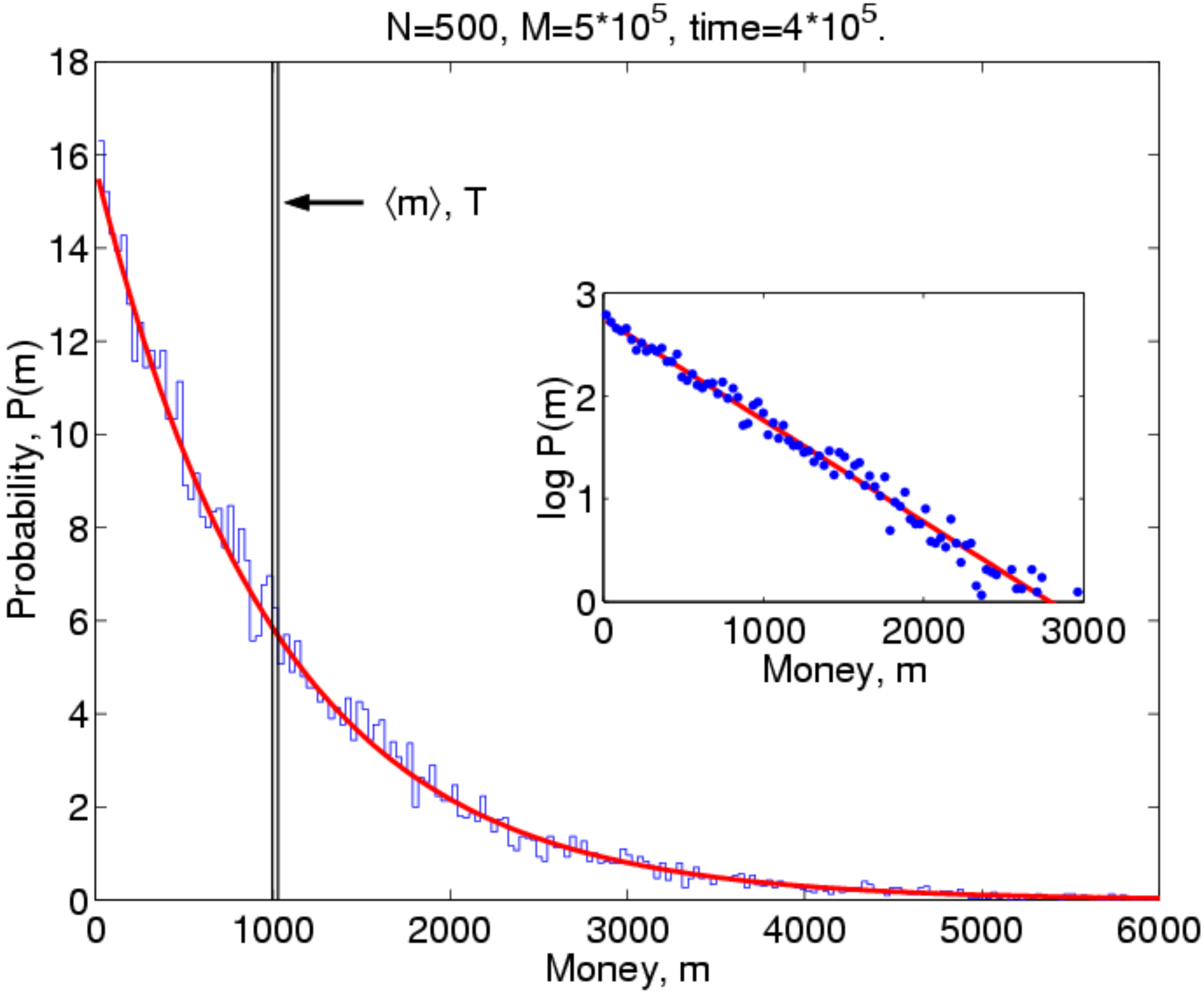
$m_1 + m_2 = m_1' + m_2'$

Detailed balance:

$w_{12\to1'2'}P(m_1)\,P(m_2) = w_{1'2'\to12}P(m_1')\,P(m_2')$

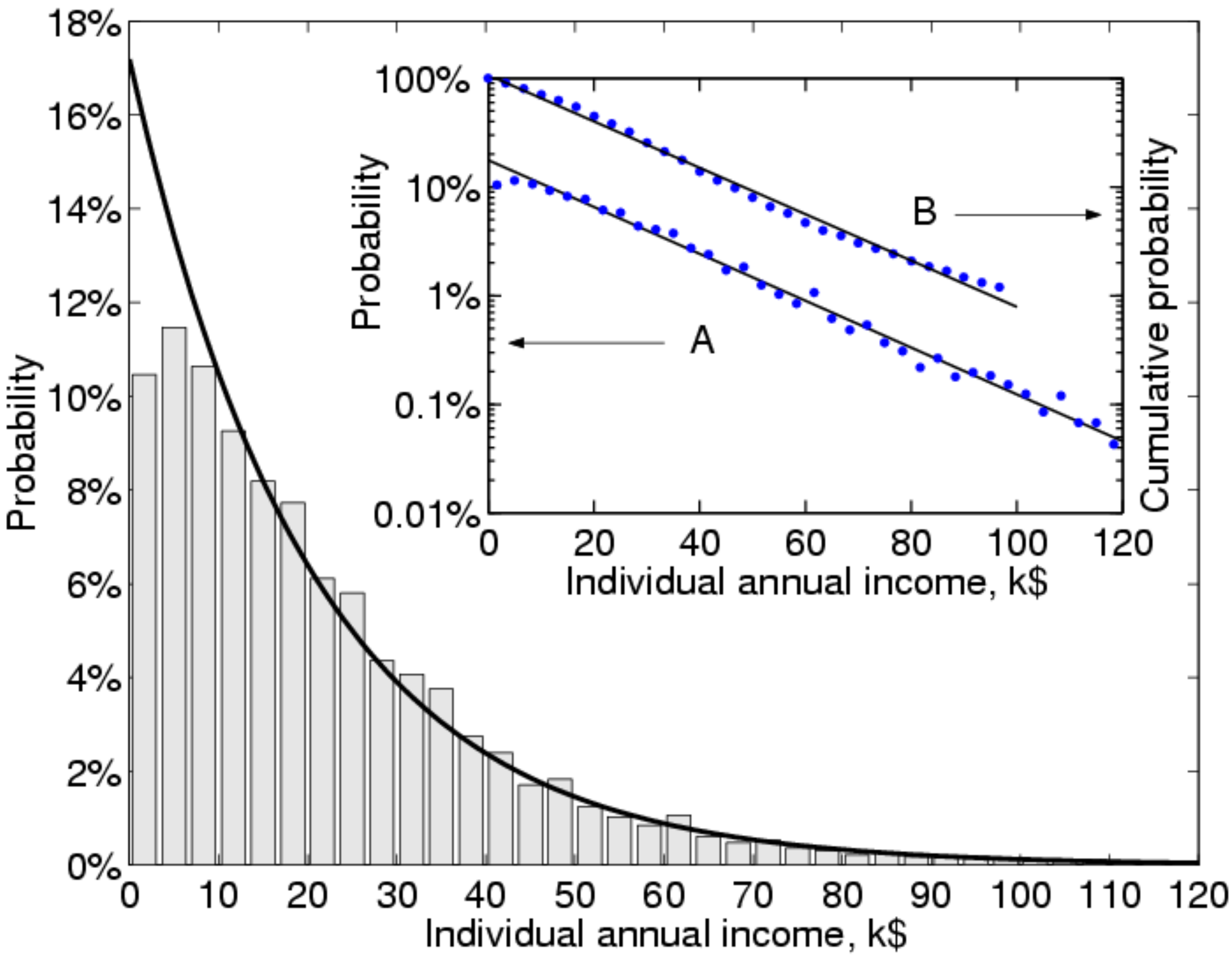
Boltzmann-Gibbs probability distribution $P(m) \propto \exp(-m/T)$ of money $m$, where $T = \langle m\rangle$ is the money temperature.

# Computer simulation of money redistribution




The stationary distribution of money *m* is exponential: $P(m) \propto e^{-m/T}$

# Probability distribution of individual income

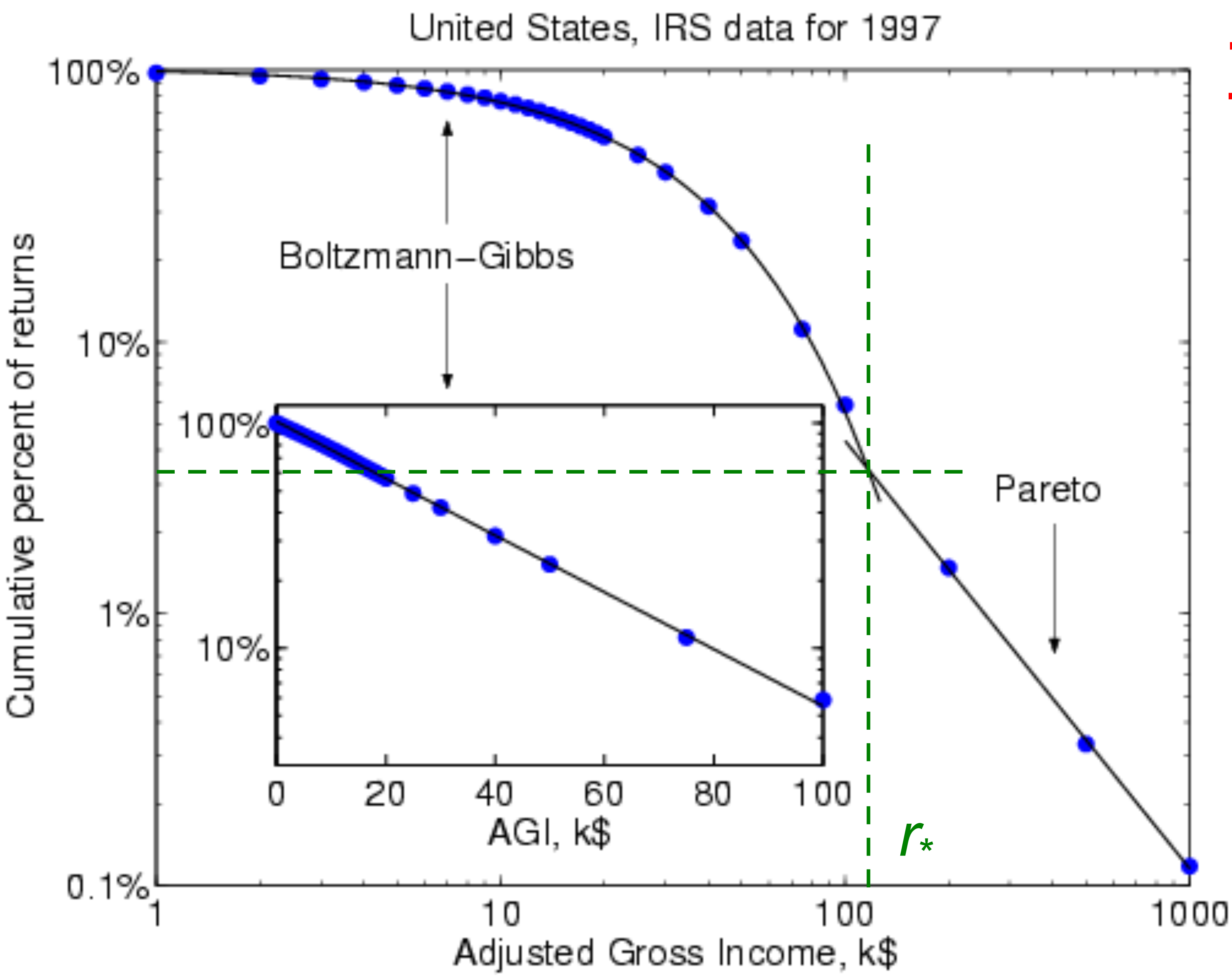



US Census data 1996 – histogram and points A

PSID: Panel Study of Income Dynamics, 1992 (U. Michigan) – points B

Distribution of income $r$ is exponential: $P(r) \propto e^{-r/T}$

# Income distribution in the USA, 1997



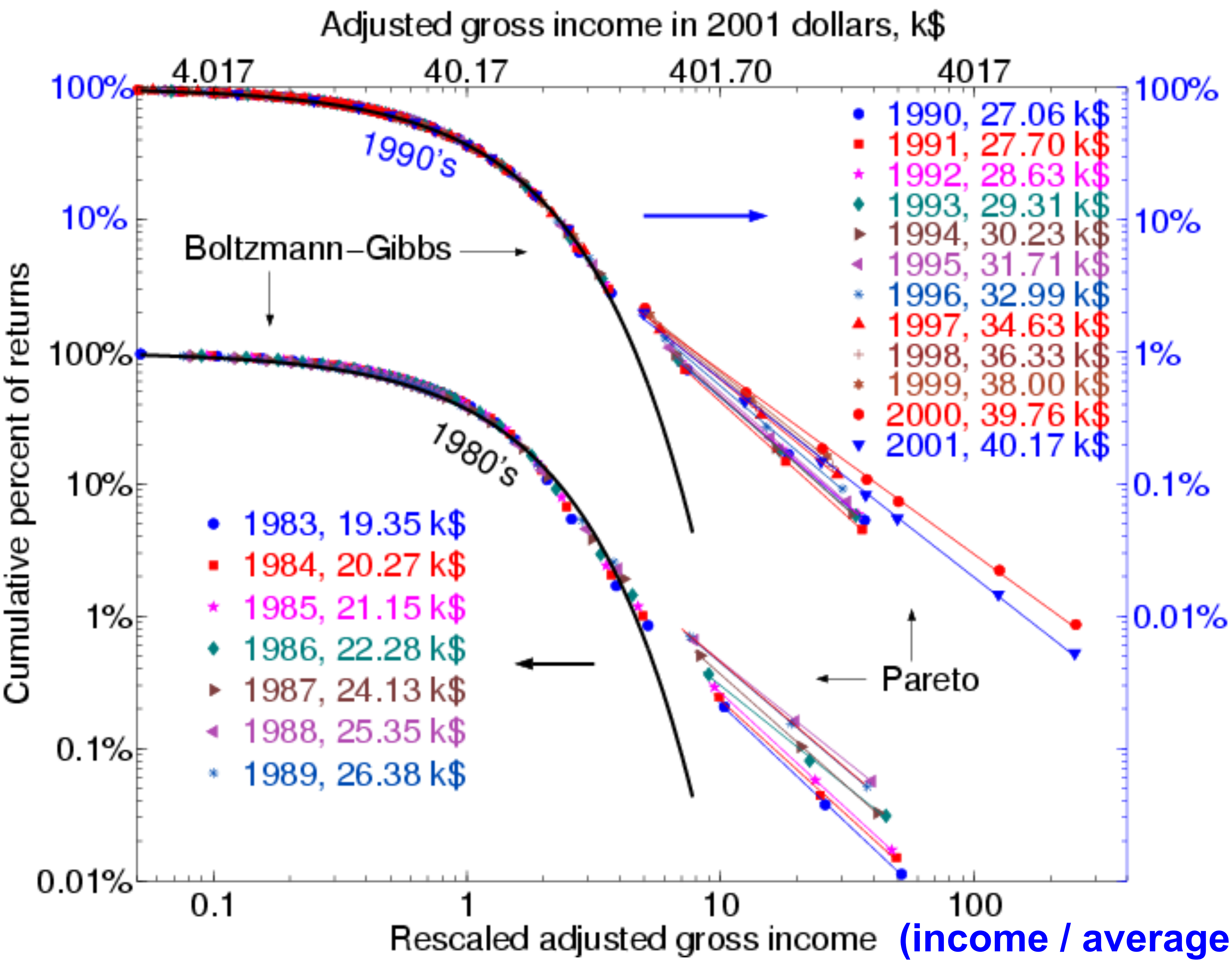

## Two-class society

**Upper Class**
- Pareto power law
- 3% of population
- 16% of income
- Income > 120 k$: investments, capital

**Lower Class**
- Boltzmann-Gibbs exponential law
- 97% of population
- 84% of income
- Income < 120 k$: wages, salaries

"Thermal" bulk and "super-thermal" tail distribution

# Income distribution in the USA, 1983-2001

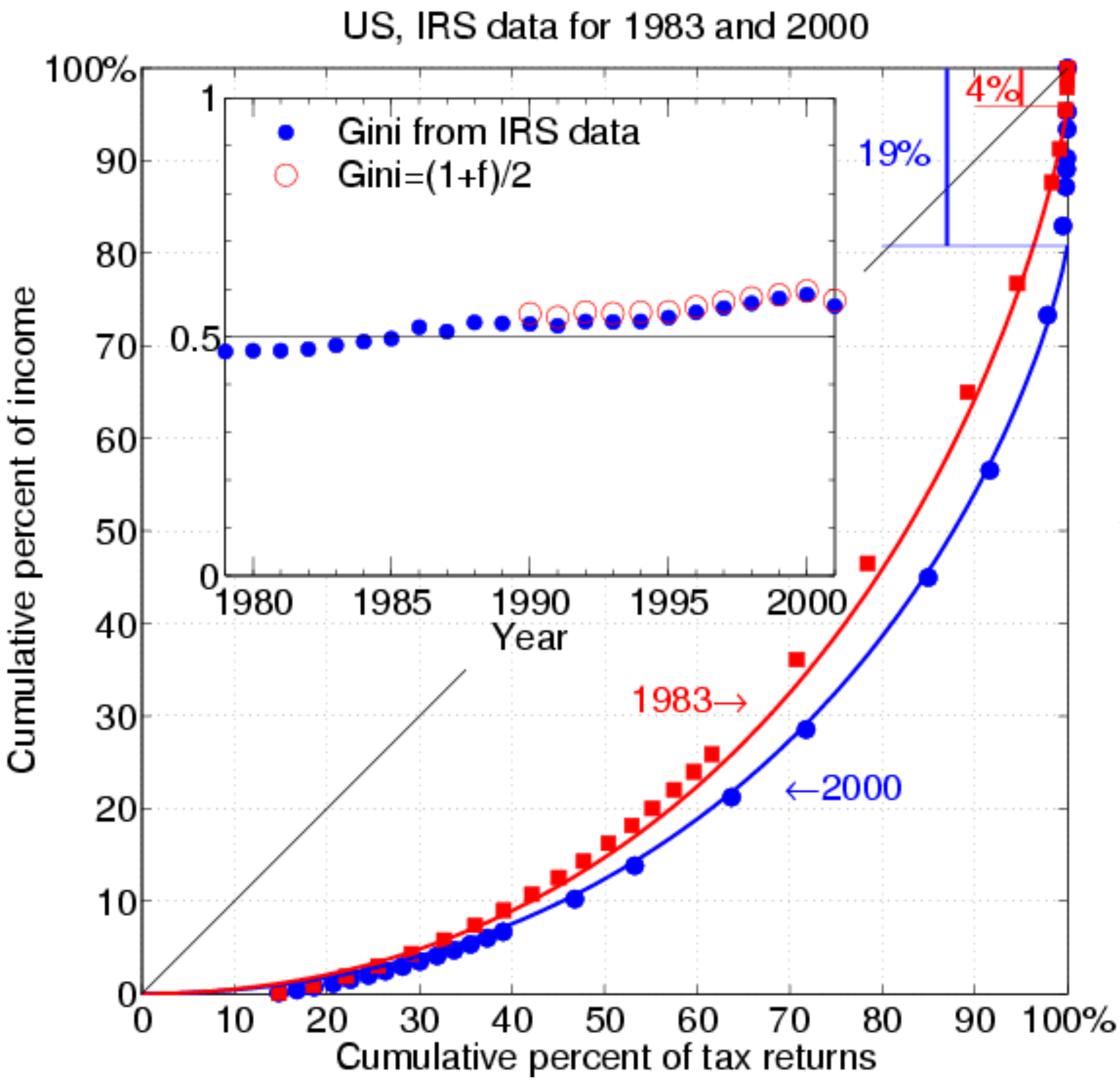



**(income / average income *T*)**

The rescaled exponential part does not change,

but the power-law part changes significantly.

# Lorenz curves and income inequality




Lorenz curve ($0<r<\infty$):

$$x(r)=\int_0^r P(r')\,dr'$$

$$y(r)=\int_0^r r'P(r')\,dr' \Big/ \langle r' \rangle$$

For exponential distribution, G=1/2 and the Lorenz curve is

$$y = x + (1-x)\ln(1-x)$$

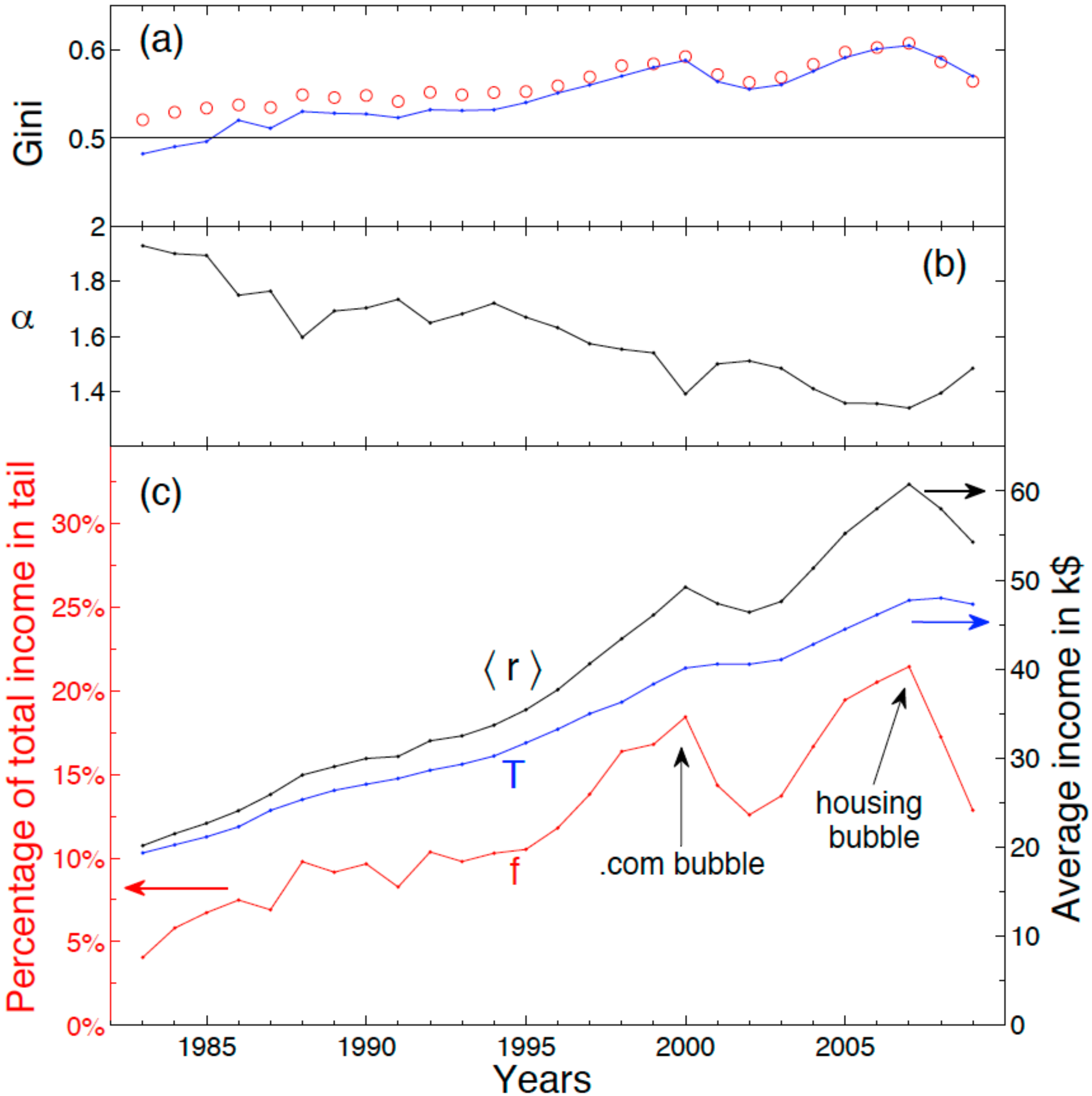
With a tail, the Lorenz curve is

$$y = (1-f)[x+(1-x)\ln(1-x)] + f\Theta(x-1),$$

where $f$ is the tail income, and Gini coefficient is $G=(1+f)/2$.

A measure of inequality, the Gini coefficient is $G = \dfrac{\text{Area(diagonal line - Lorenz curve)}}{\text{Area(Triangle beneath diagonal)}}$

# Time evolution of income inequality in USA




Gini coefficient G=(1+*f*)/2

Income inequality peaks during speculative bubbles in financial markets

$$f = \frac{\langle r \rangle - T}{\langle r \rangle}$$

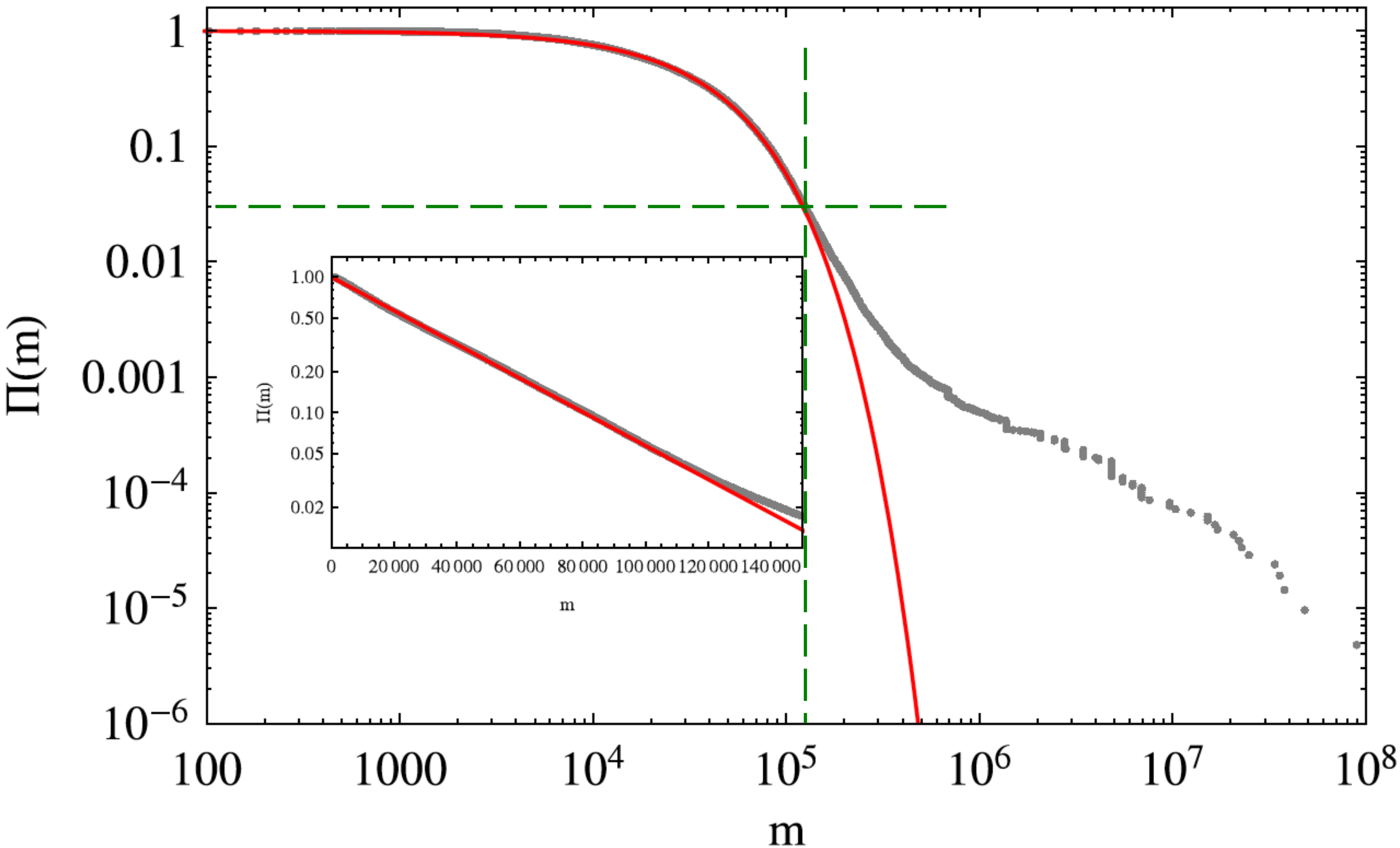
*f* - fraction of income in the tail

*<r>* – average income in the whole system

*T* – average income in the exponential part

# Income distribution in European Union, 2008

Jagielski and Kutner, *Physica A* **392**, 2130 (2013)

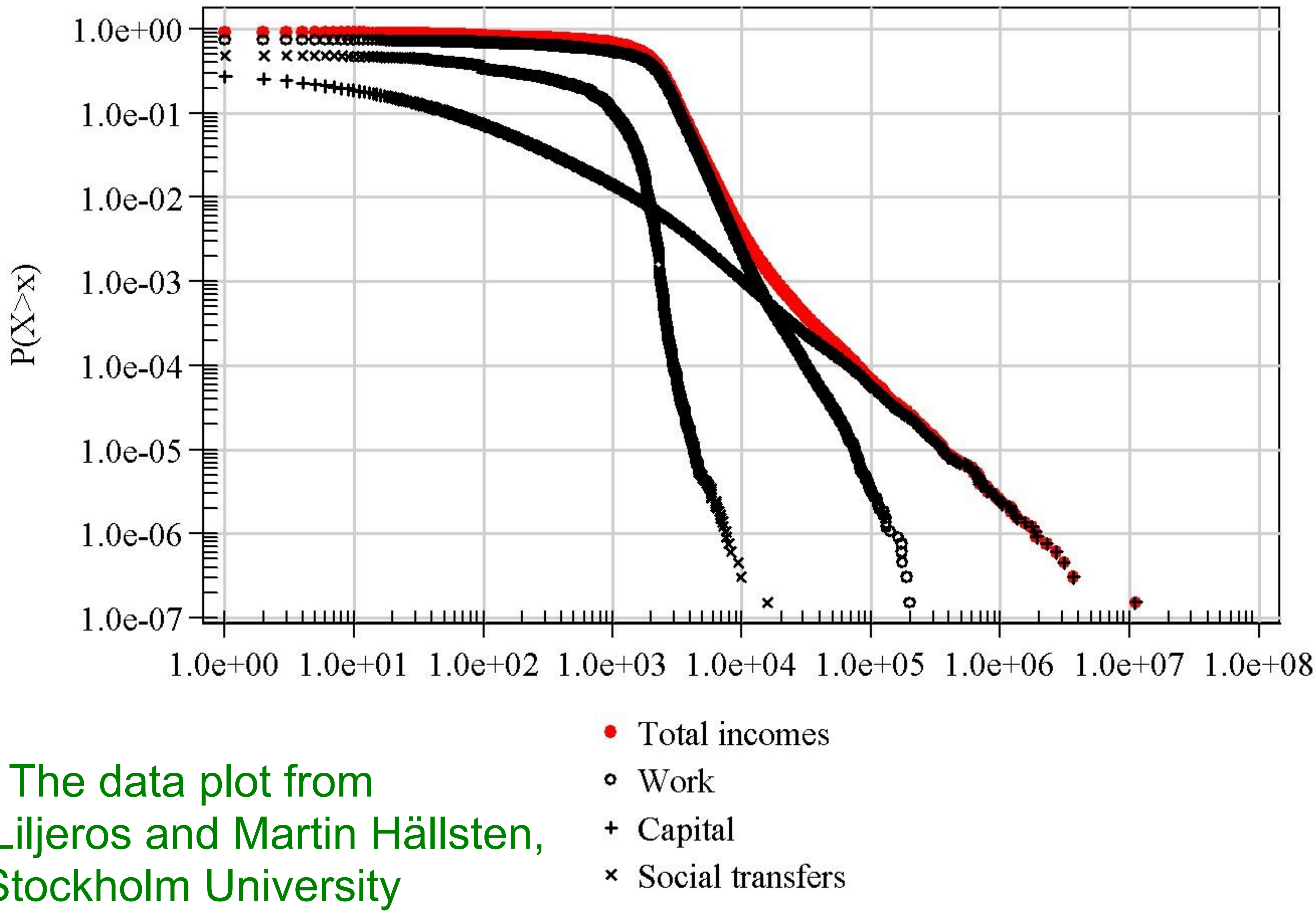



Income distribution is exponential for 97% of population.

# The origin of two classes

- Different sources of income: salaries and wages for the lower class, and capital gains and investments for the upper class.
- Their income dynamics can be described by additive and multiplicative diffusion, correspondingly.
- From the social point of view, these can be the classes of employees and employers, as described by Karl Marx.
- Emergence of classes from the initially equal agents was simulated by Ian Wright "The Social Architecture of Capitalism" *Physica A* **346**, 589 (2005), see also the book "Classical Econophysics" (2009)

# Income distribution in Sweden




The data plot from
Fredrik Liljeros and Martin Hällsten,
Stockholm University

# Diffusion model for income kinetics

Suppose income changes by small amounts $\Delta r$ over time $\Delta t$.
Then $P(r,t)$ satisfies the Fokker-Planck equation for $0<r<\infty$:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial r}\left( AP + \frac{\partial}{\partial r}(BP)\right), \quad A = -\left\langle \frac{\Delta r}{\Delta t}\right\rangle, \quad B = \left\langle \frac{(\Delta r)^2}{2\Delta t}\right\rangle.$$

For a stationary distribution, $\partial_t P = 0$ and $\frac{\partial}{\partial r}(BP) = -AP.$

For the lower class, $\Delta r$ are independent of $r$ – additive diffusion, so $A$ and $B$ are constants. Then, $P(r) \propto \exp(-r/T)$, where $T = B/A$, – an exponential distribution.

For the upper class, $\Delta r \propto r$ – multiplicative diffusion, so $A = ar$ and $B = br^2$. Then, $P(r) \propto 1/r^{\alpha+1}$, where $\alpha = 1+a/b$, – a power-law distribution.

For the upper class, income does change in percentages, as shown by Fujiwara, Souma, Aoyama, Kaizoji, and Aoki (2003) for the tax data in Japan. For the lower class, the data is not known yet.

# Additive and multiplicative income diffusion

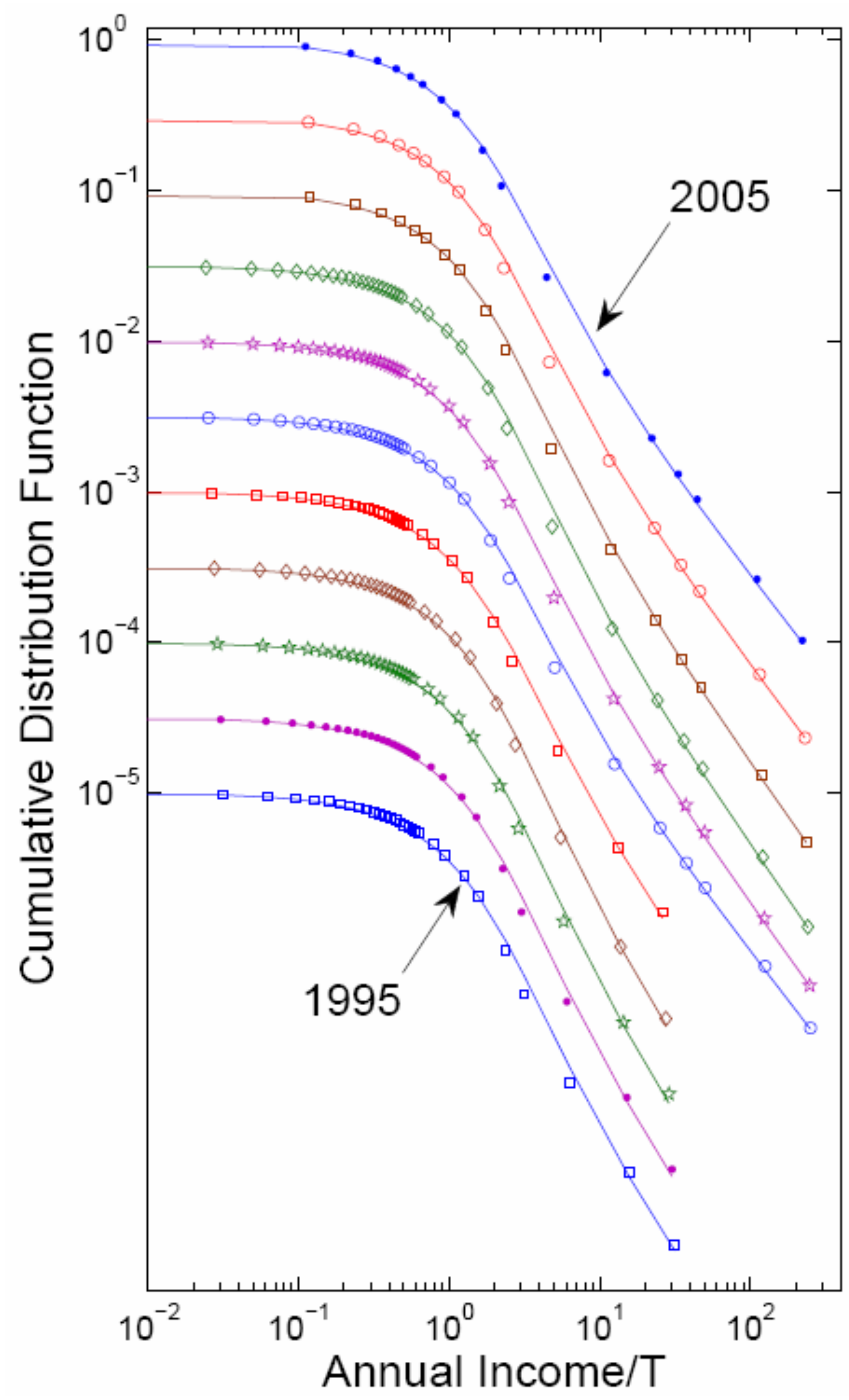
If the additive and multiplicative diffusion processes are present simultaneously, then $A = A_0 + ar$ and $B = B_0 + br^2 = b(r_0^2 + r^2)$. The stationary solution of the FP equation is

$$P(r) = \frac{C\, e^{-\frac{r_0}{T}\arctan\left(\frac{r}{r_0}\right)}}{\left[1+(r/r_0)^2\right]^{1+a/2b}}$$

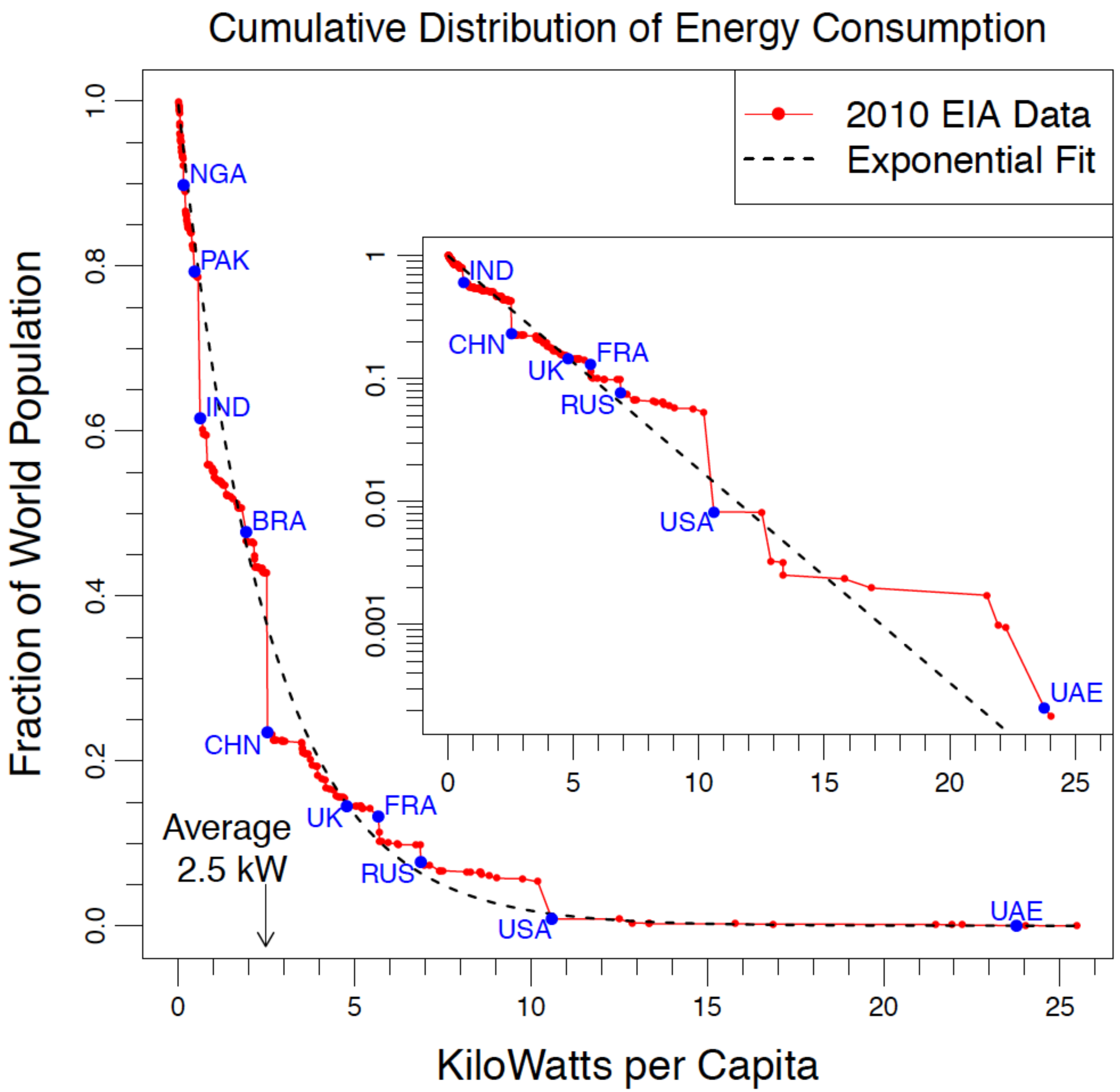
It interpolates between the exponential and the power-law distributions and has 3 parameters:

- $T = B_0/A_0$ – temperature of the exponential part
- $\alpha = 1 + a/b$ – power-law exponent of the upper tail
- $r_0$ – crossover income between the lower and upper parts.

Banerjee & Yakovenko, *RMP* (2009), NJP (2010)
Fiaschi & Marsili, *JEBO* (2012)
Karl Pearson, *Proc. Roy. Soc. London* (1895)



# Global inequality in energy consumption

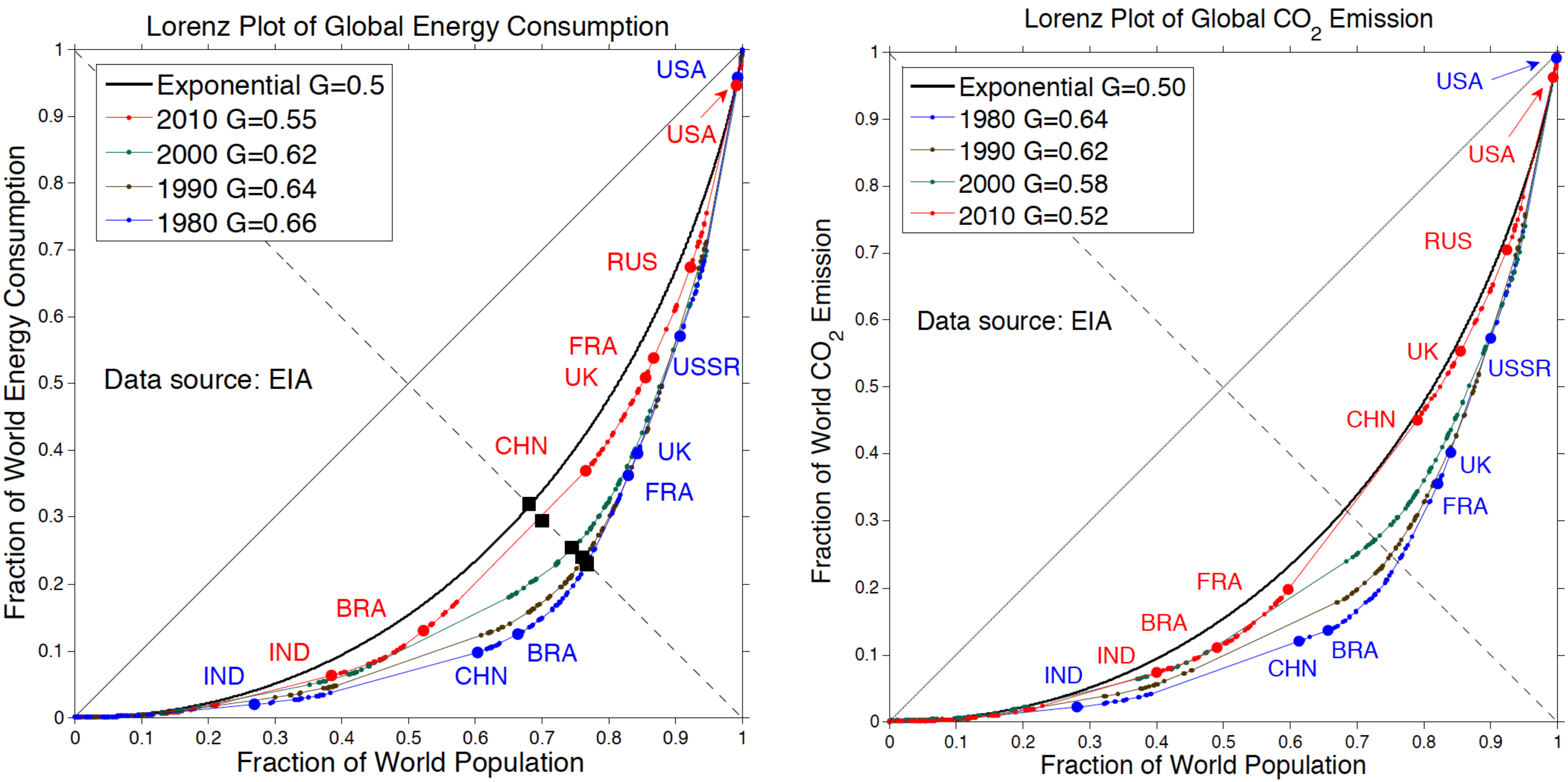



Global distribution of energy consumption per person is roughly exponential.

Division of a limited resource + entropy maximization produce exponential distribution.

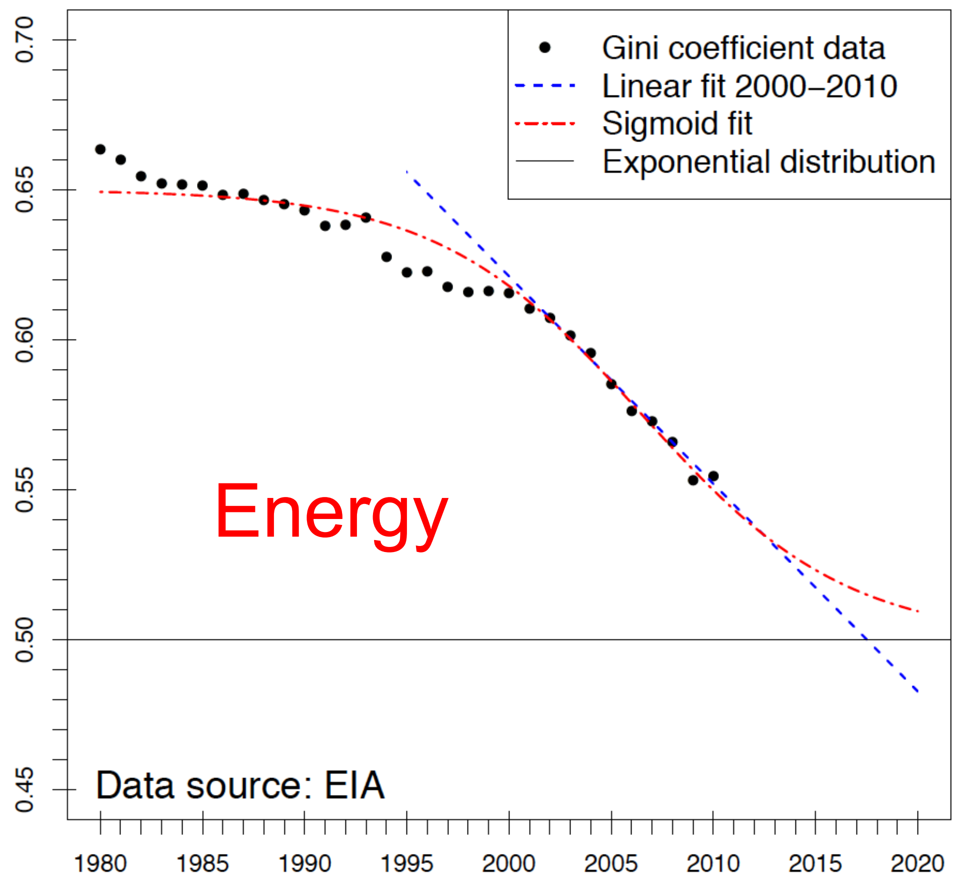
Physiological energy consumption of a human at rest is about 100 W

# Global inequality in energy consumption

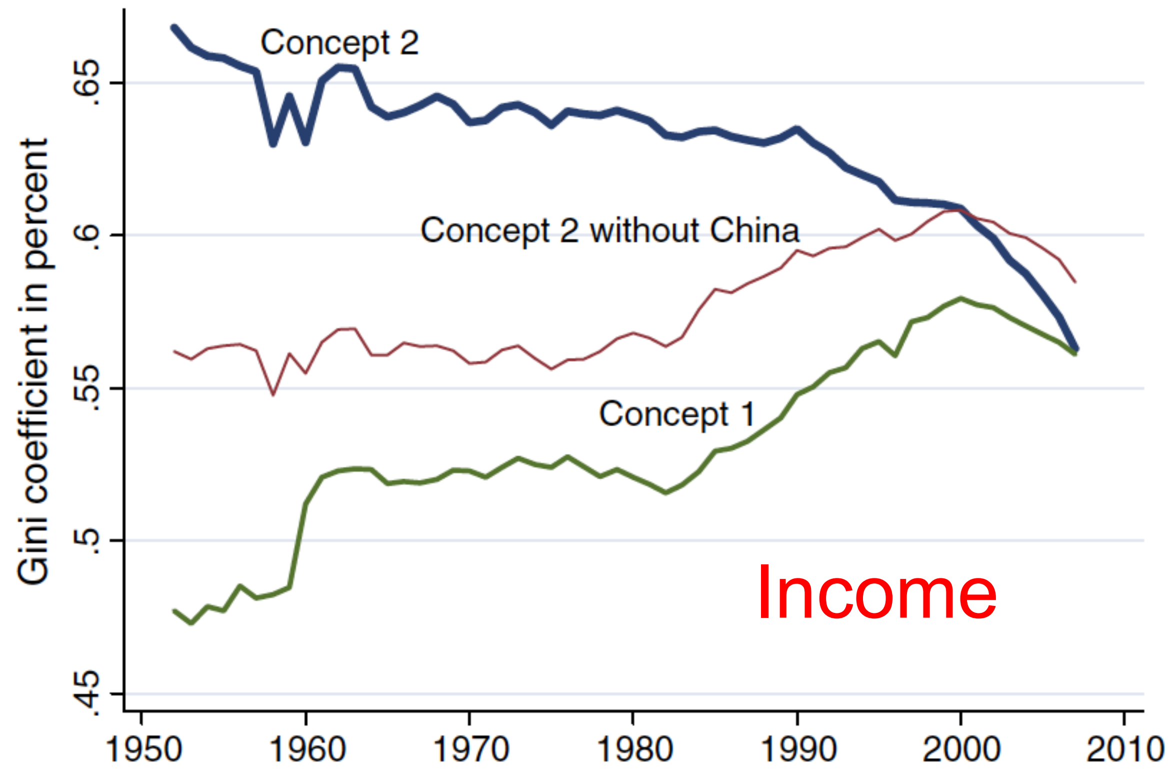



- Global inequality in energy consumption decreases.
- Energy consumption evolves toward the exponential distribution.
- The law of 1/3: Top 1/3 of world population consumes 2/3 of energy

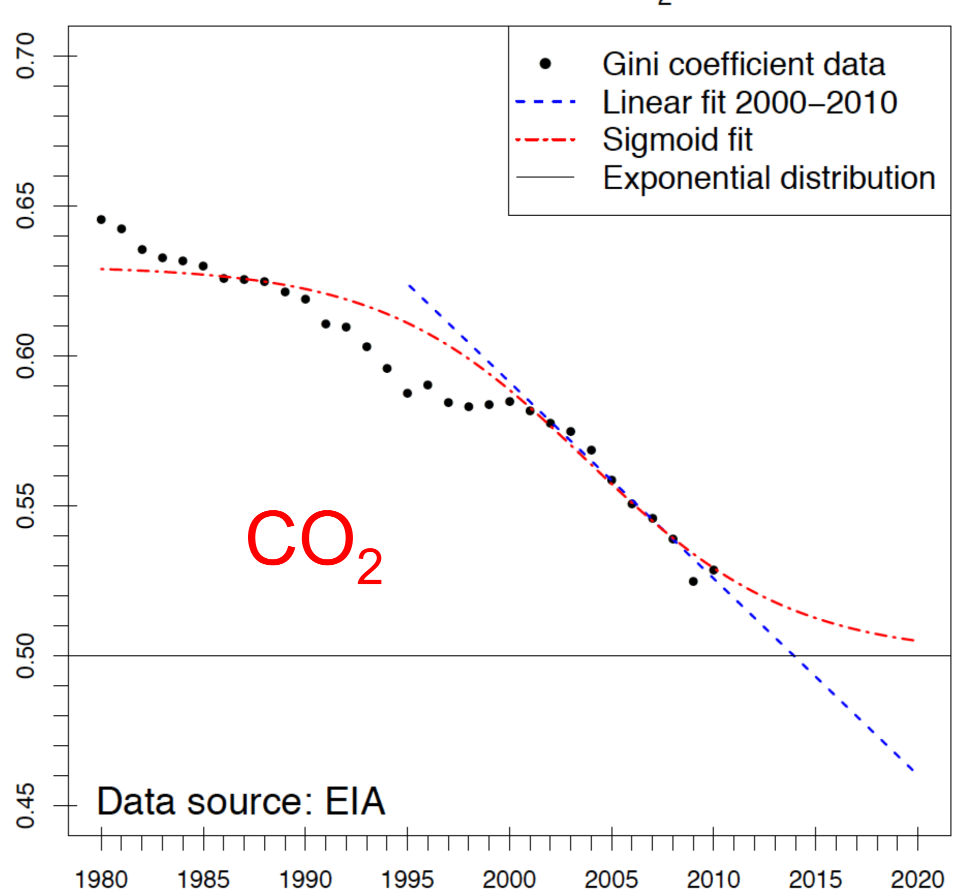
Gini Coefficient of Global Energy Consumption




B. Milanovic, *J Econ Inequal* **10**, 1 (2012)

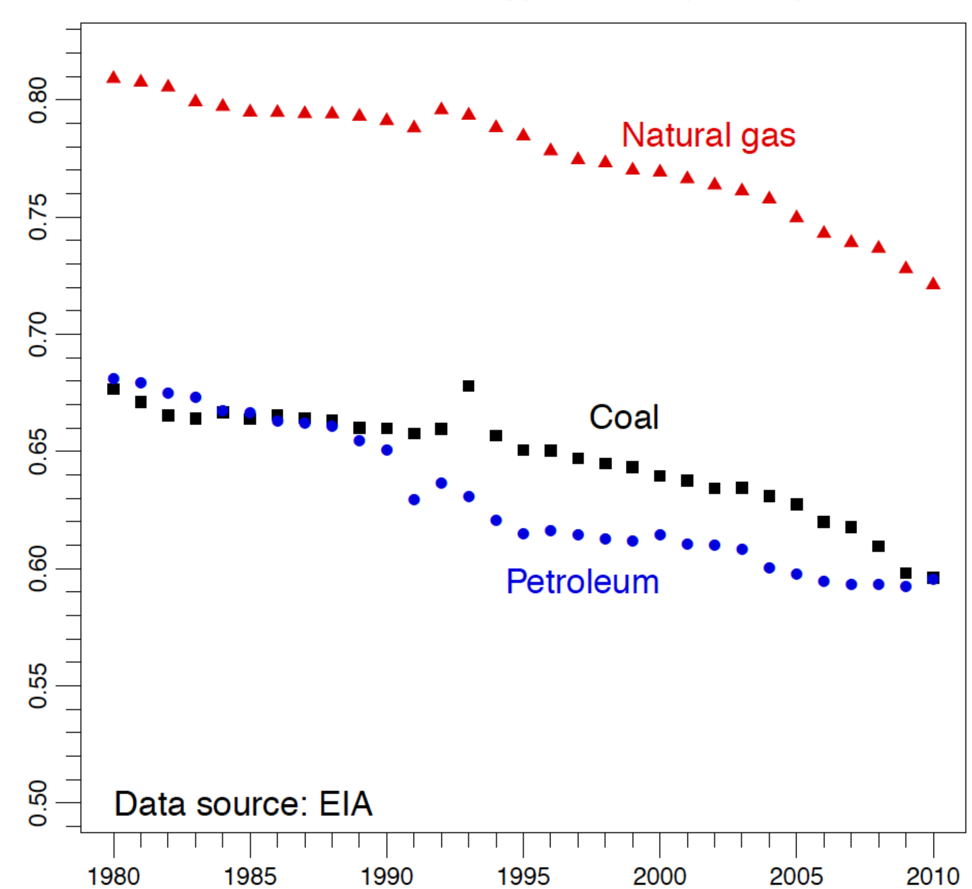



Gini Coefficient of Global $CO_2$ Emission




Money
=
Energy
=
Carbon

Gini Coefficients of Energy Consumption by Source



# Conclusions

- The probability distribution of money is stable and has an equilibrium only when a boundary condition, such as $m>0$, is imposed.
- When debt is permitted, the distribution of money becomes unstable, unless some sort of a limit on maximal debt is imposed.
- Income distribution in the USA has a two-class structure: exponential ("thermal") for the great majority (97-99%) of population and power-law ("superthermal") for the top 1-3% of population.
- The exponential part of the distribution is very stable and does not change in time, except for a slow increase of temperature $T$ (the average income).
- The power-law tail is not universal and was increasing significantly for the last 20 years. It peaked and crashed in 2000 and 2007 with the speculative bubbles in financial markets.
- The global distribution of energy consumption per person is highly unequal and roughly exponential. This inequality is important in dealing with the global energy problems.
- All papers at **http://physics.umd.edu/~yakovenk/econophysics/**