

Statistical Mechanics of Money, Income, Debt, and Energy Consumption

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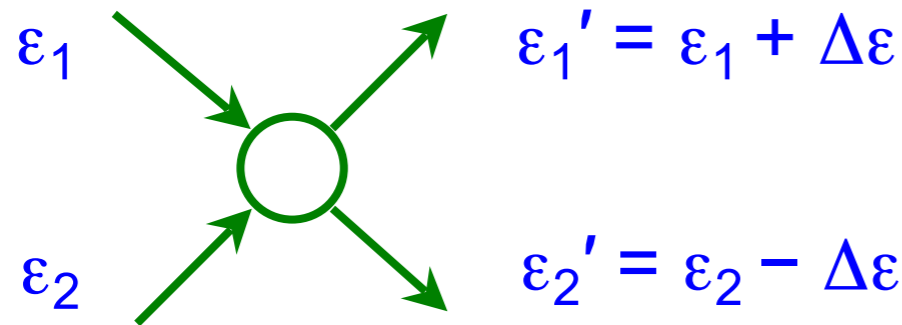
- *European Physical Journal B* **17**, 723 (2000) > >
- *Reviews of Modern Physics* **81**, 1703 (2009)
- Book *Classical Econophysics* (Routledge, 2009)
- *Entropy* **15**, 5565 (2013).

- Outline:**
- Statistical mechanics of money
 - Debt and financial instability
 - Two-class structure of income distribution
 - Global inequality in energy consumption

INET funding 2013

Boltzmann-Gibbs probability distribution of **money**

Collisions between atoms



Conservation of energy:

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_1' + \varepsilon_2'$$

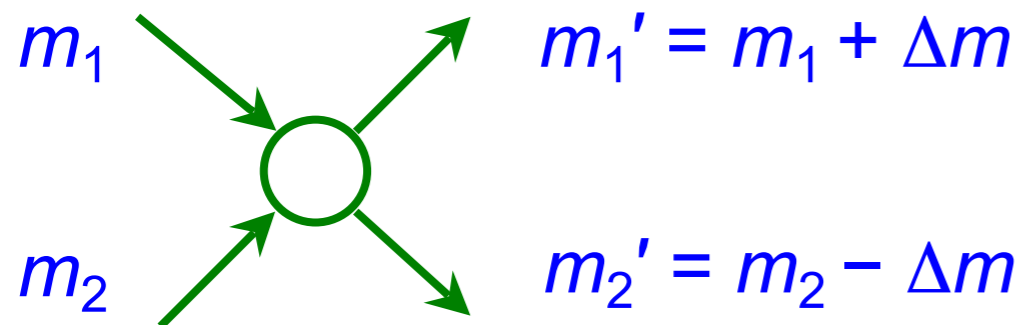
Detailed balance:

~~$$w_{12 \rightarrow 1'2'} P(\varepsilon_1) P(\varepsilon_2) = w_{1'2' \rightarrow 12} P(\varepsilon_1') P(\varepsilon_2')$$~~

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$ of energy ε , where $T = \langle \varepsilon \rangle$ is temperature. It is **universal** – independent of model rules, provided the model belongs to the **time-reversal symmetry** class.

Boltzmann-Gibbs distribution **maximizes entropy** $S = -\sum_{\varepsilon} P(\varepsilon) \ln P(\varepsilon)$ under the constraint of conservation law $\sum_{\varepsilon} P(\varepsilon) \varepsilon = \text{const.}$

Economic transactions between agents



Conservation of money:

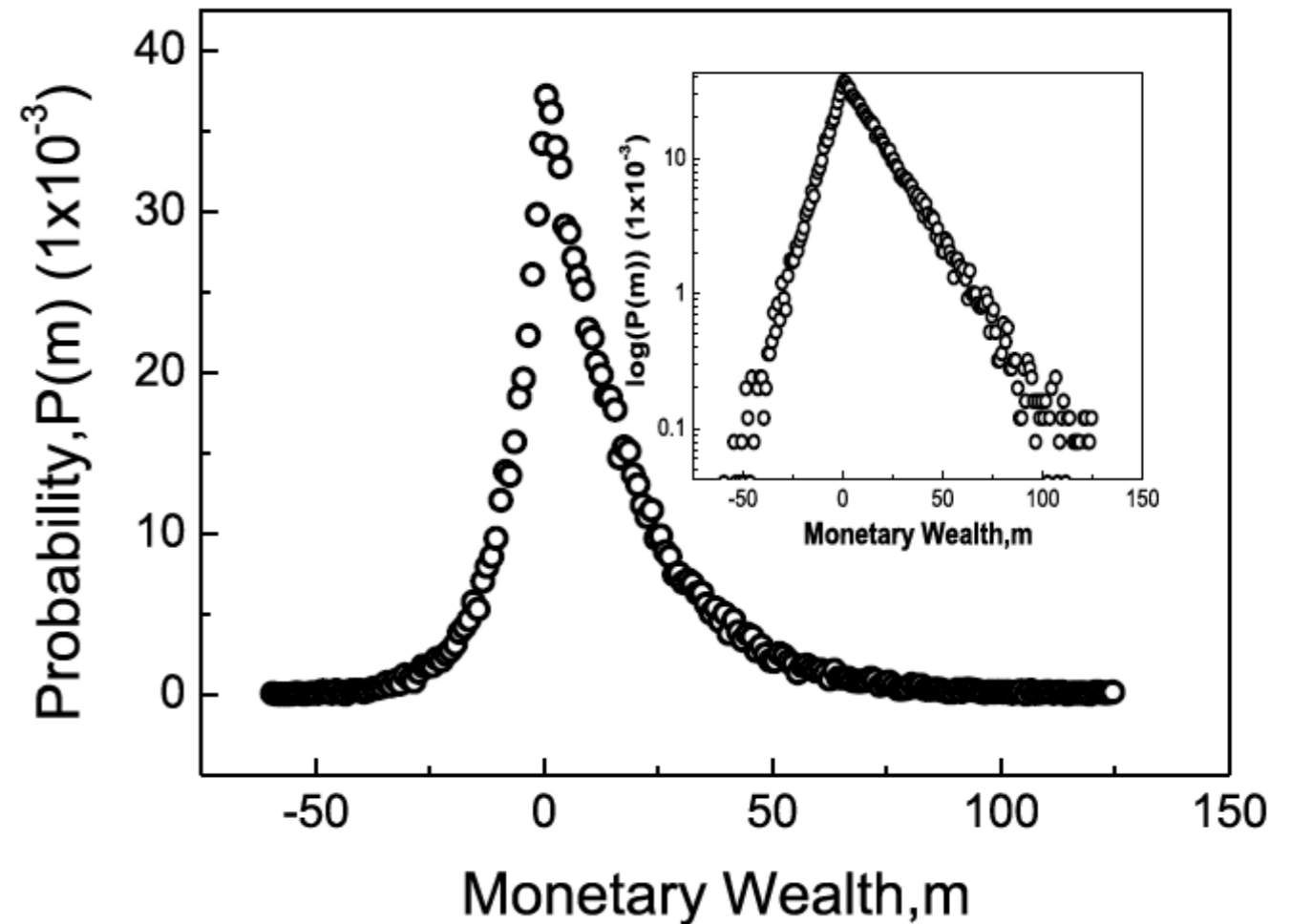
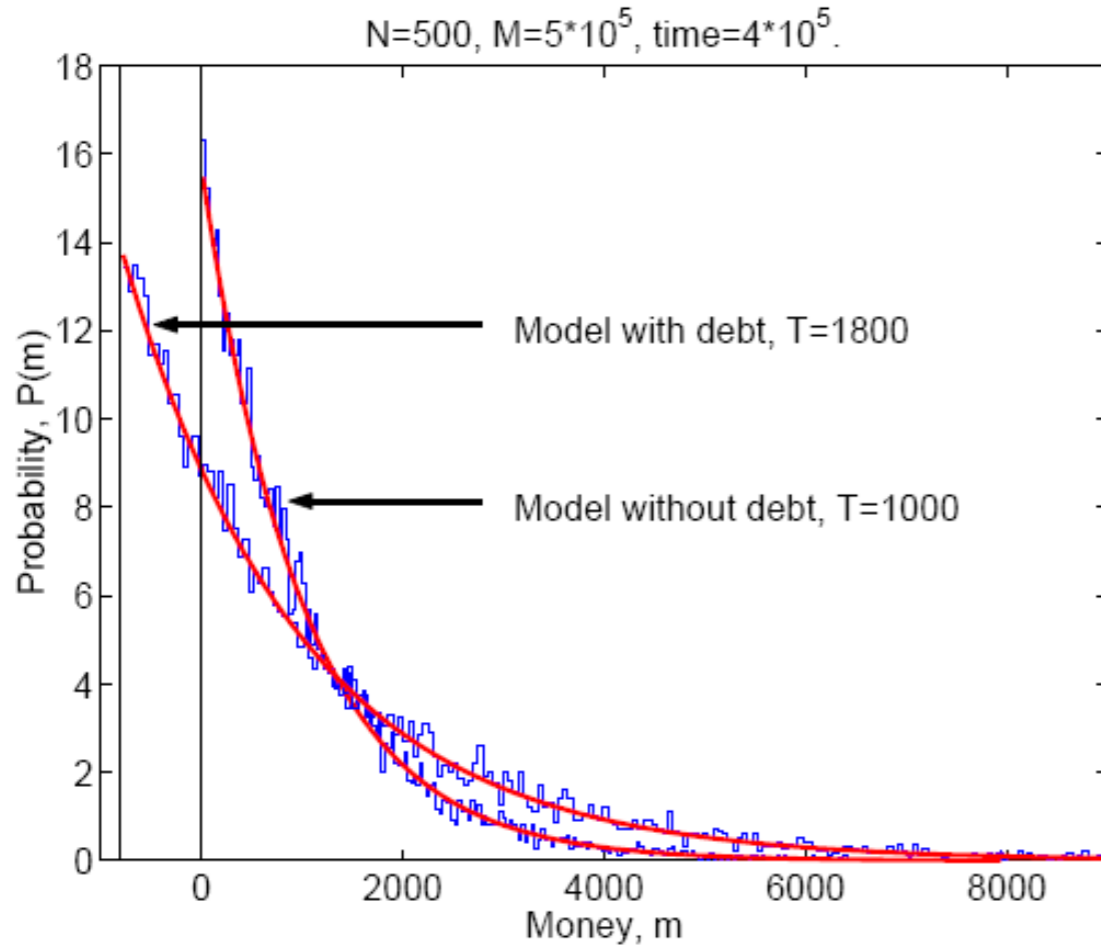
$$m_1 + m_2 = m_1' + m_2'$$

Detailed balance:

~~$$w_{12 \rightarrow 1'2'} P(m_1) P(m_2) = w_{1'2' \rightarrow 12} P(m_1') P(m_2')$$~~

Boltzmann-Gibbs probability distribution $P(m) \propto \exp(-m/T)$ of **money** m , where $T = \langle m \rangle$ is the **money temperature**.

Money distribution with debt



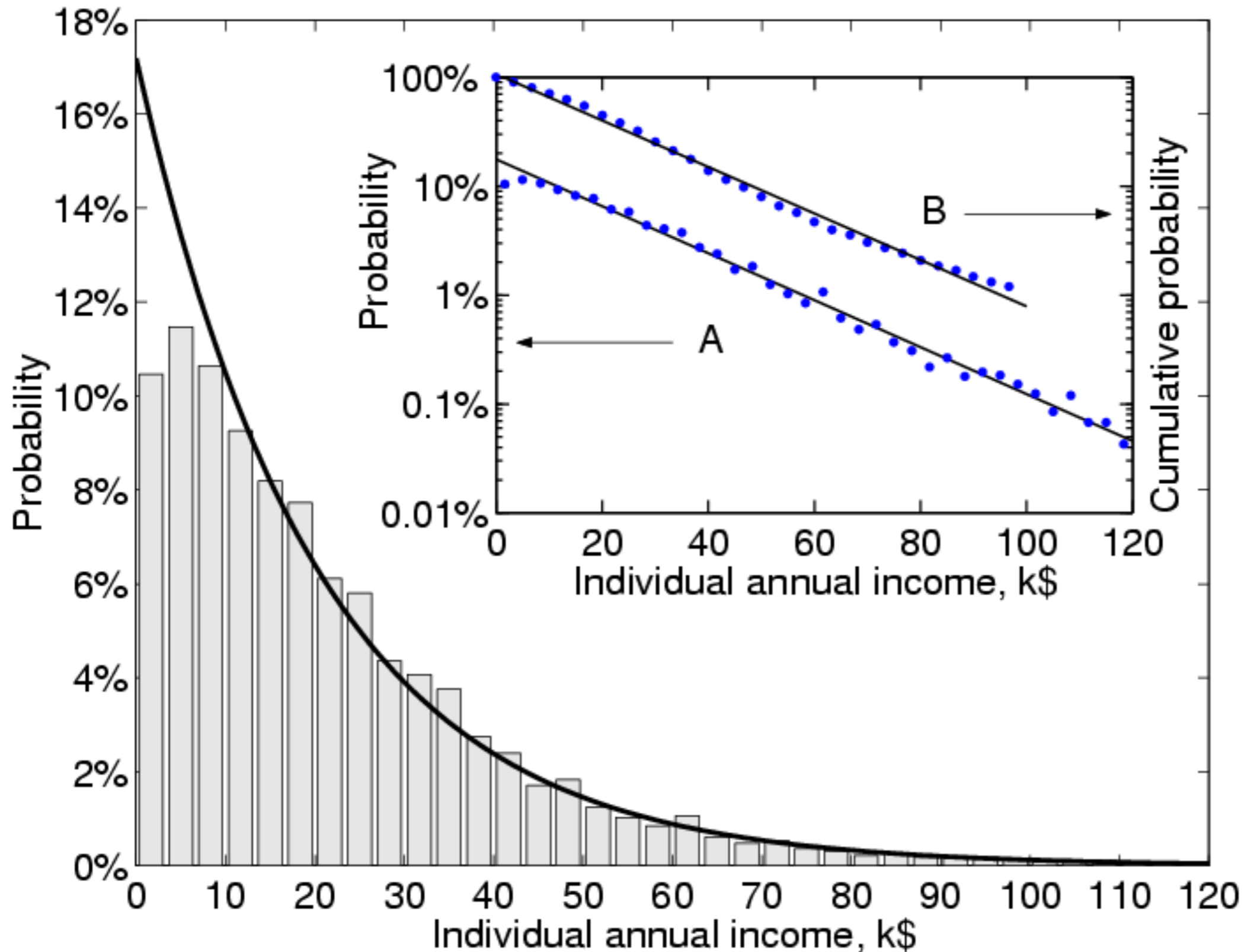
Debt per person is limited to 800 units.

Total debt in the system is limited via the Required Reserve Ratio (RRR):

Xi, Ding, Wang, Physica A 357, 543 (2005)

- In practice, RRR is enforced inconsistently and does not limit total debt.
- Without a constraint on debt, the system does **not have a stationary equilibrium**.
- Free market itself does not have an intrinsic mechanism for limiting debt, and there is **no such thing as the equilibrium debt**.

Probability distribution of individual income

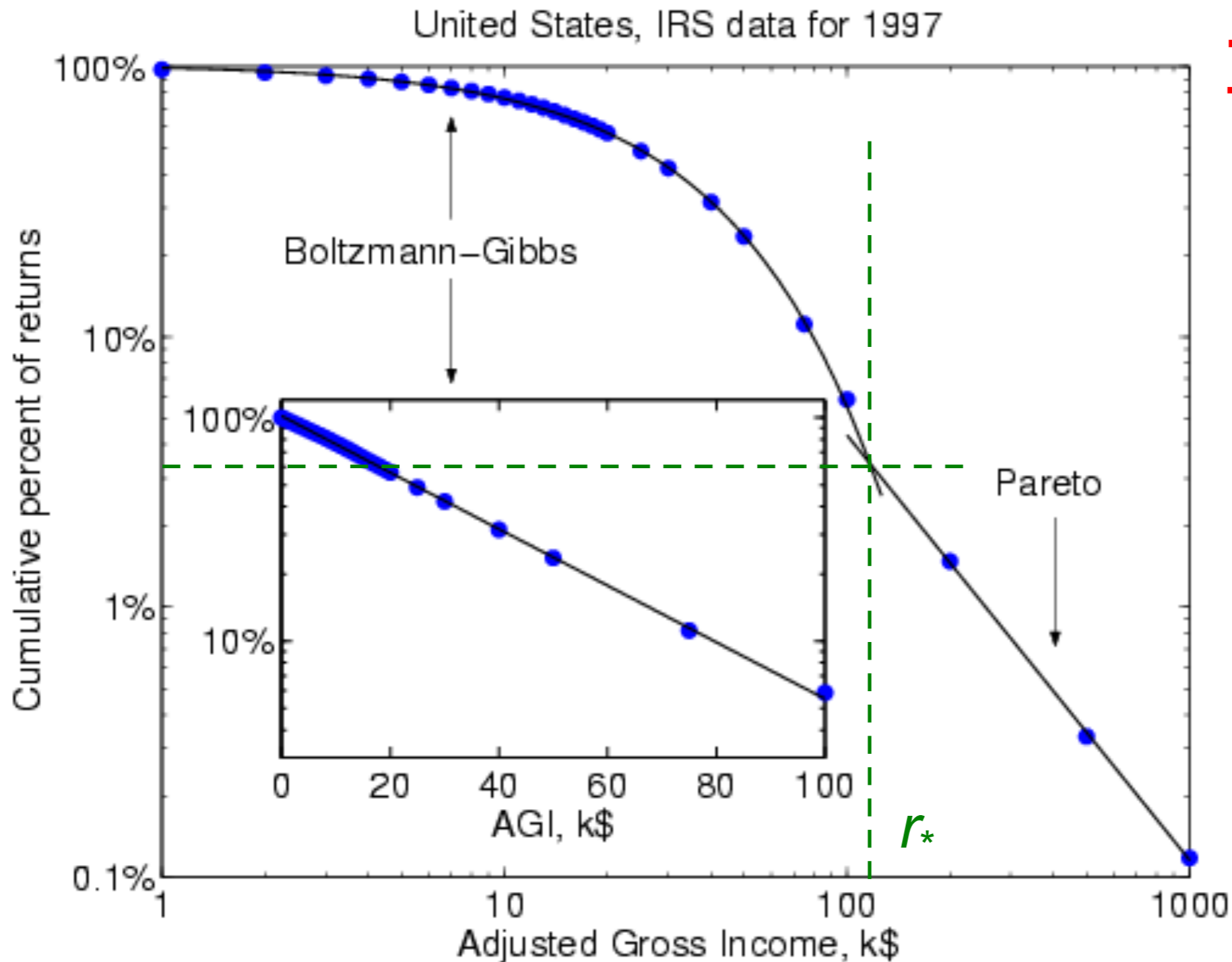


US Census
data 1996 –
histogram and
points A

PSID: Panel
Study of Income
Dynamics, 1992
(U. Michigan) –
points B

Distribution
of income r
is exponential:
 $P(r) \propto e^{-r/T}$

Income distribution in the USA, 1997



Two-class society

Upper Class

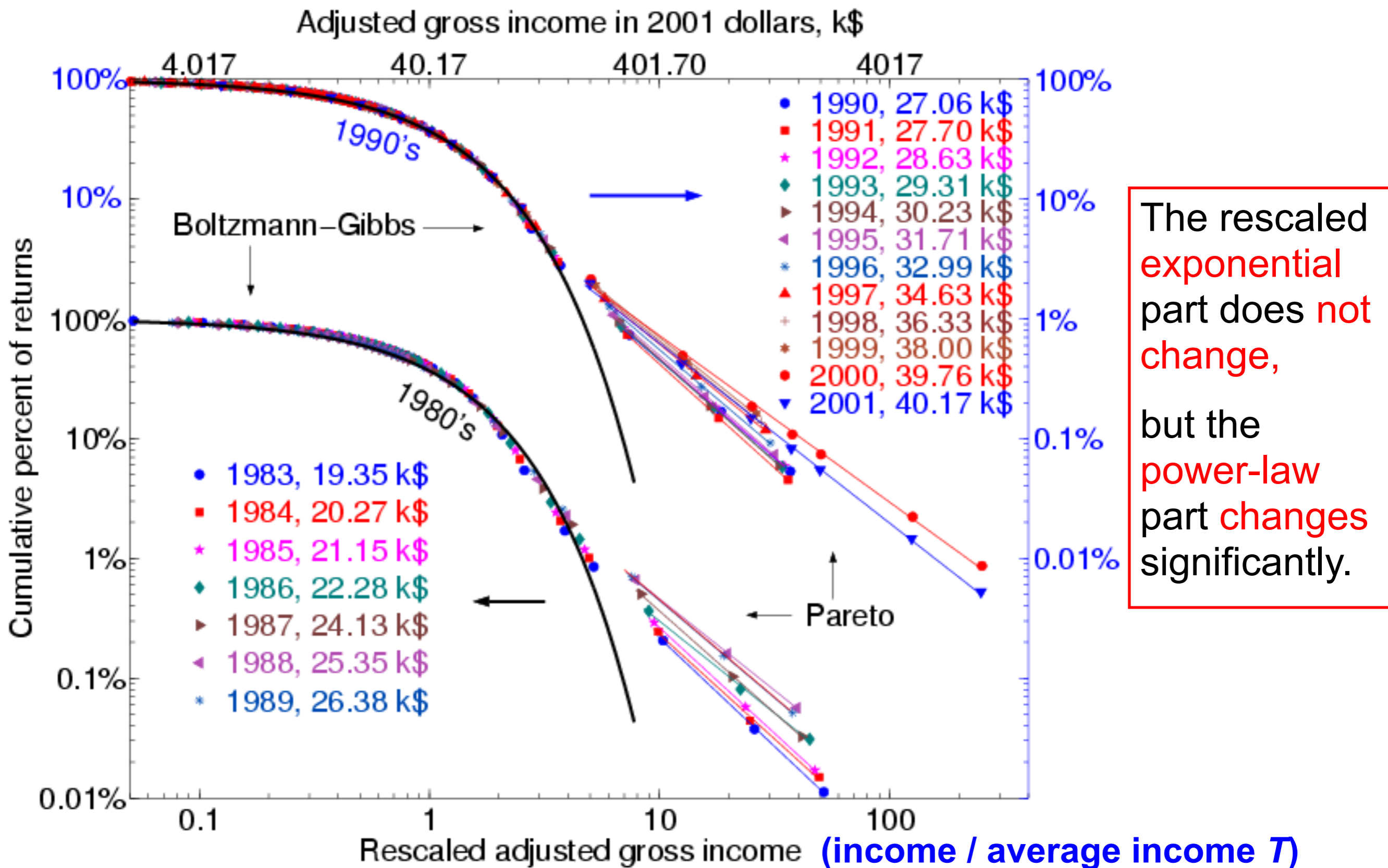
- Pareto power law
- 3% of population
- 16% of income
- Income > 120 k\$: investments, capital

Lower Class

- Boltzmann-Gibbs exponential law
- 97% of population
- 84% of income
- Income < 120 k\$: wages, salaries

“Thermal” bulk and “super-thermal” tail distribution

Income distribution in the USA, 1983-2001



Lorenz curves and income inequality

Lorenz curve ($0 < r < \infty$):

$$x(r) = \int_0^r P(r') dr'$$

$$y(r) = \int_0^r r' P(r') dr' / \langle r' \rangle$$

For exponential distribution, $G=1/2$ and the Lorenz curve is

$$y = x + (1 - x) \ln(1 - x)$$

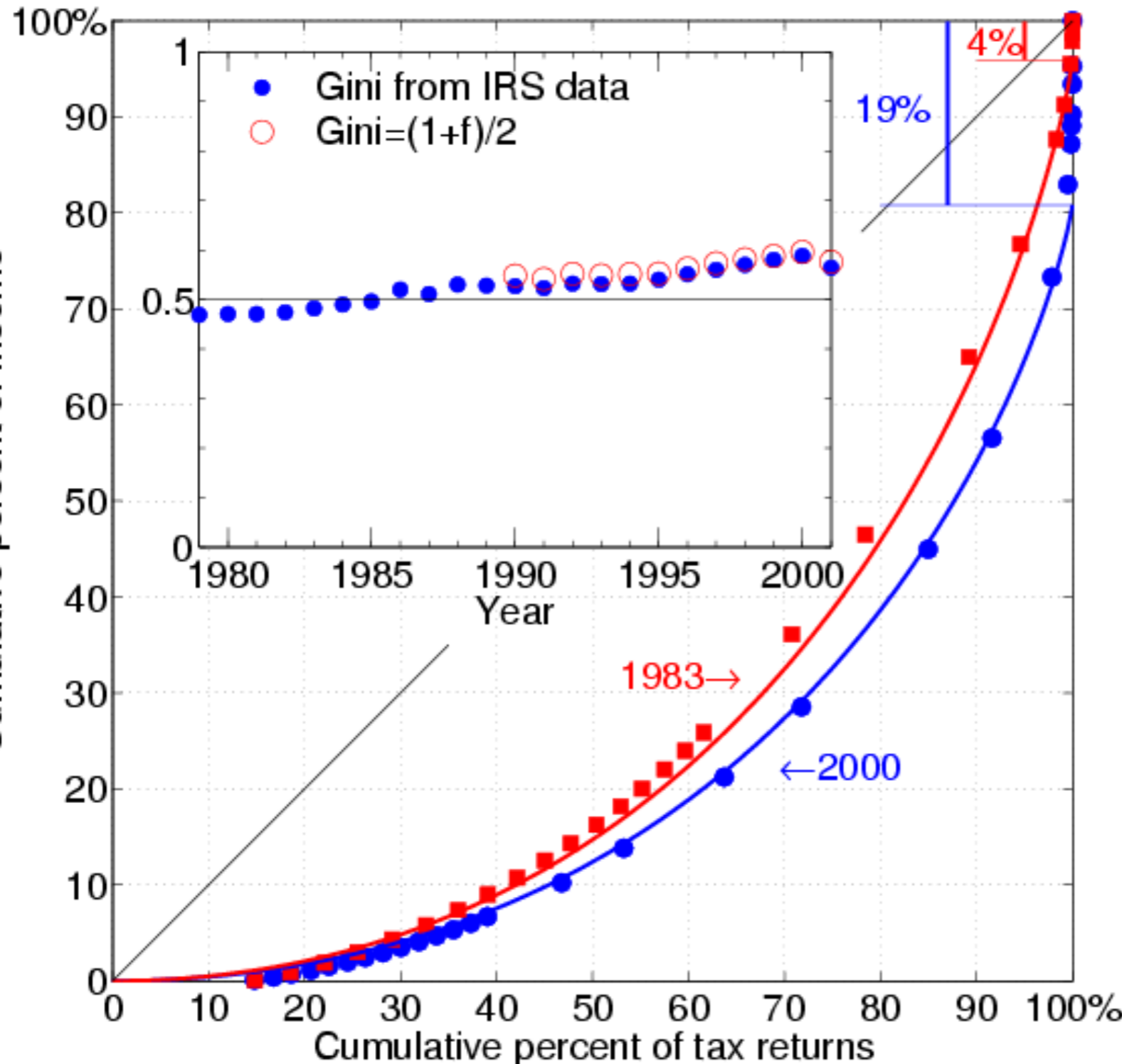
With a tail, the Lorenz curve is

$$y = (1 - f)[x + (1 - x) \ln(1 - x)] + f \Theta(x - 1),$$

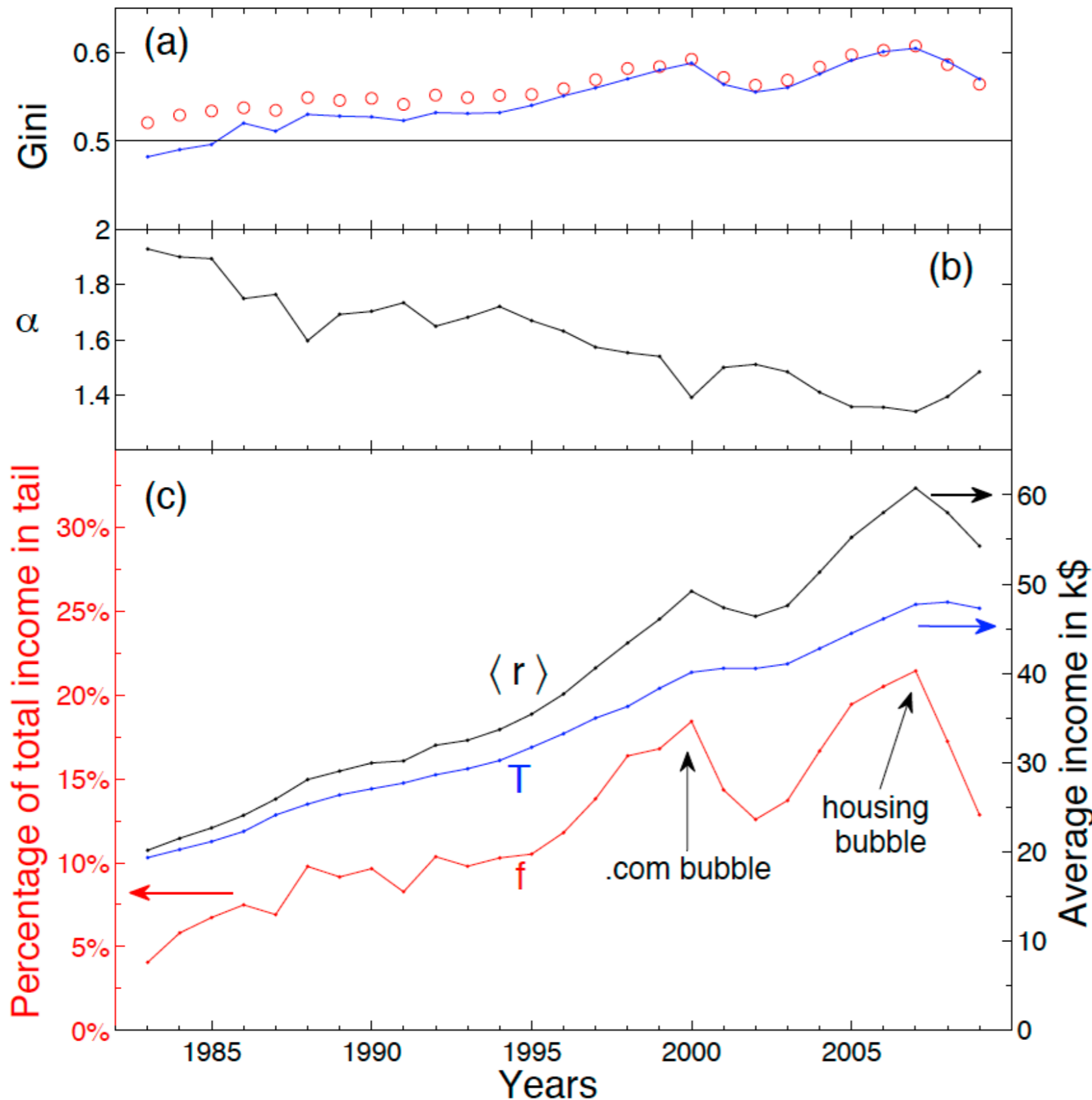
where f is the tail income, and Gini coefficient is $G=(1+f)/2$.

A measure of inequality, the Gini coefficient is $G = \frac{\text{Area}(\text{diagonal line} - \text{Lorenz curve})}{\text{Area}(\text{Triangle beneath diagonal})}$

US, IRS data for 1983 and 2000



Time evolution of income inequality in USA



Gini coefficient $G=(1+f)/2$

Income inequality peaks during speculative bubbles in financial markets

$$f = \frac{\langle r \rangle - T}{\langle r \rangle}$$

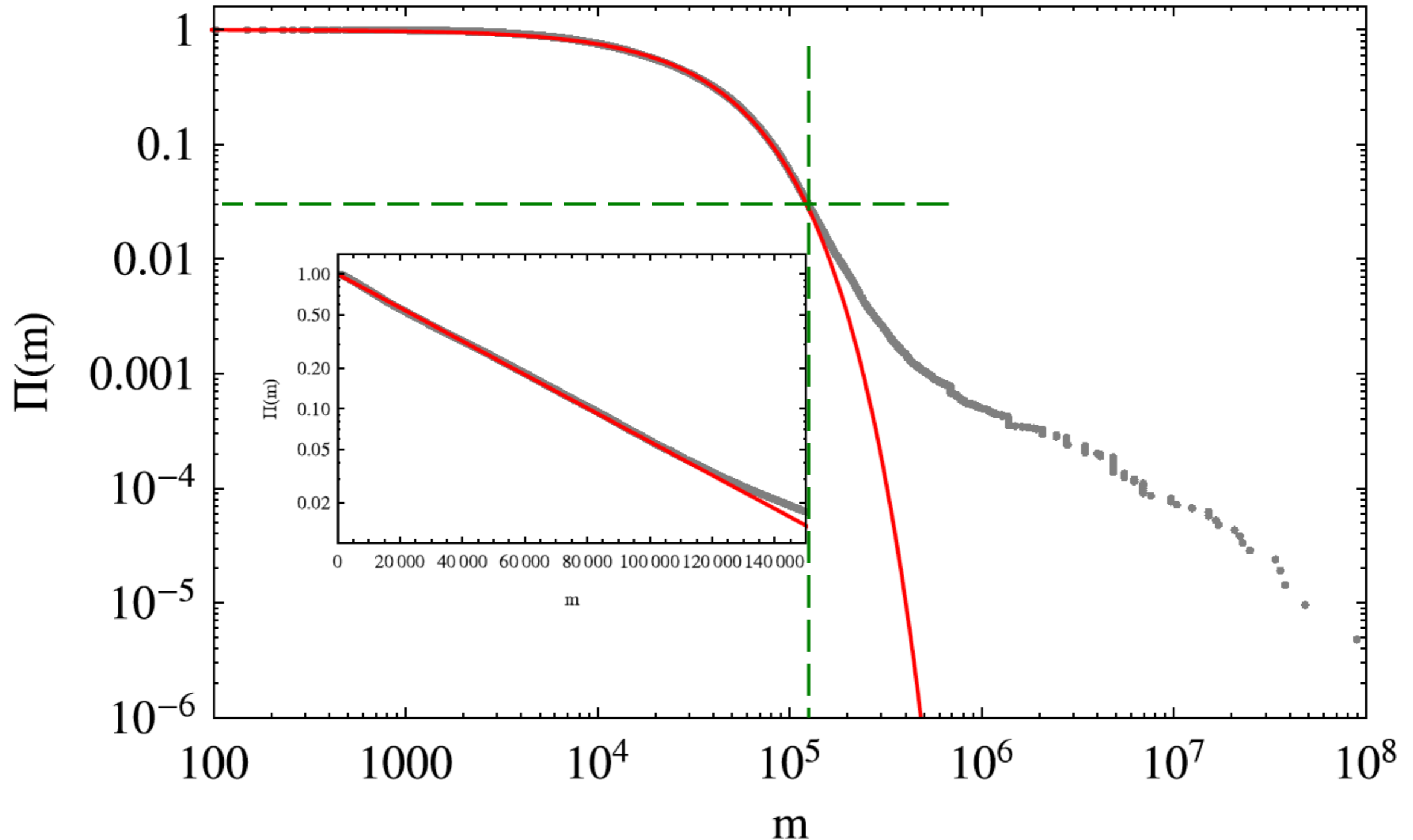
f - fraction of income in the tail

$\langle r \rangle$ - average income in the whole system

T - average income in the exponential part

Income distribution in European Union, 2008

Jagielski and Kutner, *Physica A* 392, 2130 (2013)

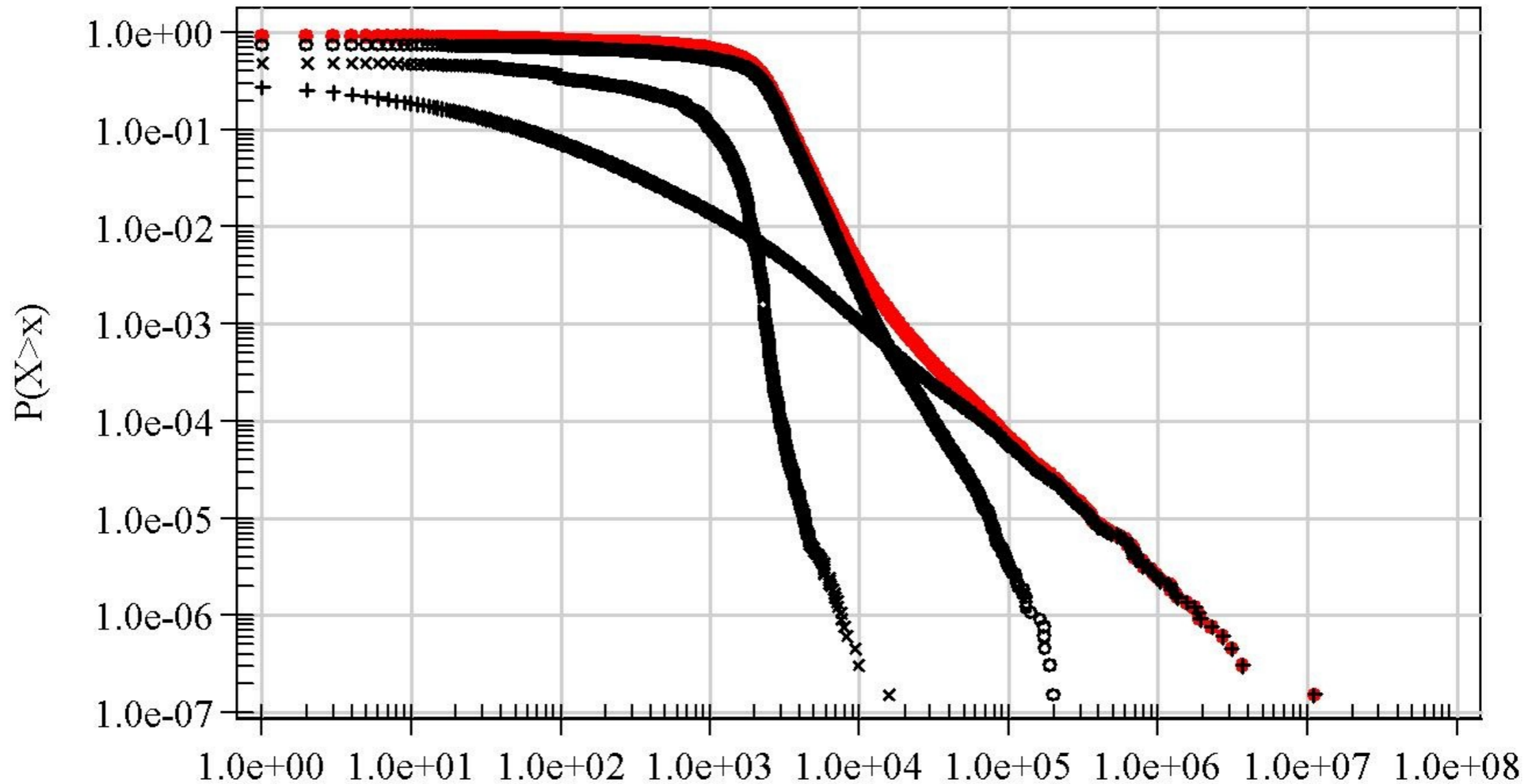


Income distribution is **exponential** for **97%** of population.

The origin of two classes

- Different sources of income: salaries and wages for the lower class, and capital gains and investments for the upper class.
- Their income dynamics can be described by additive and multiplicative diffusion, correspondingly.
- From the social point of view, these can be the classes of employees and employers, as described by Karl Marx.
- Emergence of classes from the initially equal agents was simulated by Ian Wright “The Social Architecture of Capitalism” *Physica A* 346, 589 (2005), see also the book “Classical Econophysics” (2009)

Income distribution in Sweden



The data plot from
Fredrik Liljeros and Martin Hällsten,
Stockholm University

- Total incomes
- Work
- + Capital
- × Social transfers

Diffusion model for income kinetics

Suppose income changes by small amounts Δr over time Δt .
Then $P(r,t)$ satisfies the **Fokker-Planck equation** for $0 < r < \infty$:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial r} \left(AP + \frac{\partial}{\partial r} (BP) \right), \quad A = - \left\langle \frac{\Delta r}{\Delta t} \right\rangle, \quad B = \left\langle \frac{(\Delta r)^2}{2\Delta t} \right\rangle.$$

For a stationary distribution, $\partial_t P = 0$ and $\frac{\partial}{\partial r} (BP) = -AP$.

For the **lower class**, Δr are independent of r – **additive diffusion**, so A and B are constants. Then, $P(r) \propto \exp(-r/T)$, where $T = B/A$, – **an exponential distribution**.

For the **upper class**, $\Delta r \propto r$ – **multiplicative diffusion**, so $A = ar$ and $B = br^2$.
Then, $P(r) \propto 1/r^{\alpha+1}$, where $\alpha = 1+a/b$, – **a power-law distribution**.

For the **upper class**, income does change in **percentages**, as shown by **Fujiwara, Souma, Aoyama, Kaizoji, and Aoki (2003)** for the tax data in Japan.
For the **lower class**, the data is not known yet.

Additive and multiplicative income diffusion

If the **additive** and **multiplicative** diffusion processes are present **simultaneously**, then $A = A_0 + ar$ and $B = B_0 + br^2 = b(r_0^2 + r^2)$. The stationary solution of the FP equation is

$$P(r) = \frac{C e^{-\frac{r_0}{T} \arctan\left(\frac{r}{r_0}\right)}}{\left[1 + (r/r_0)^2\right]^{1+a/2b}}$$

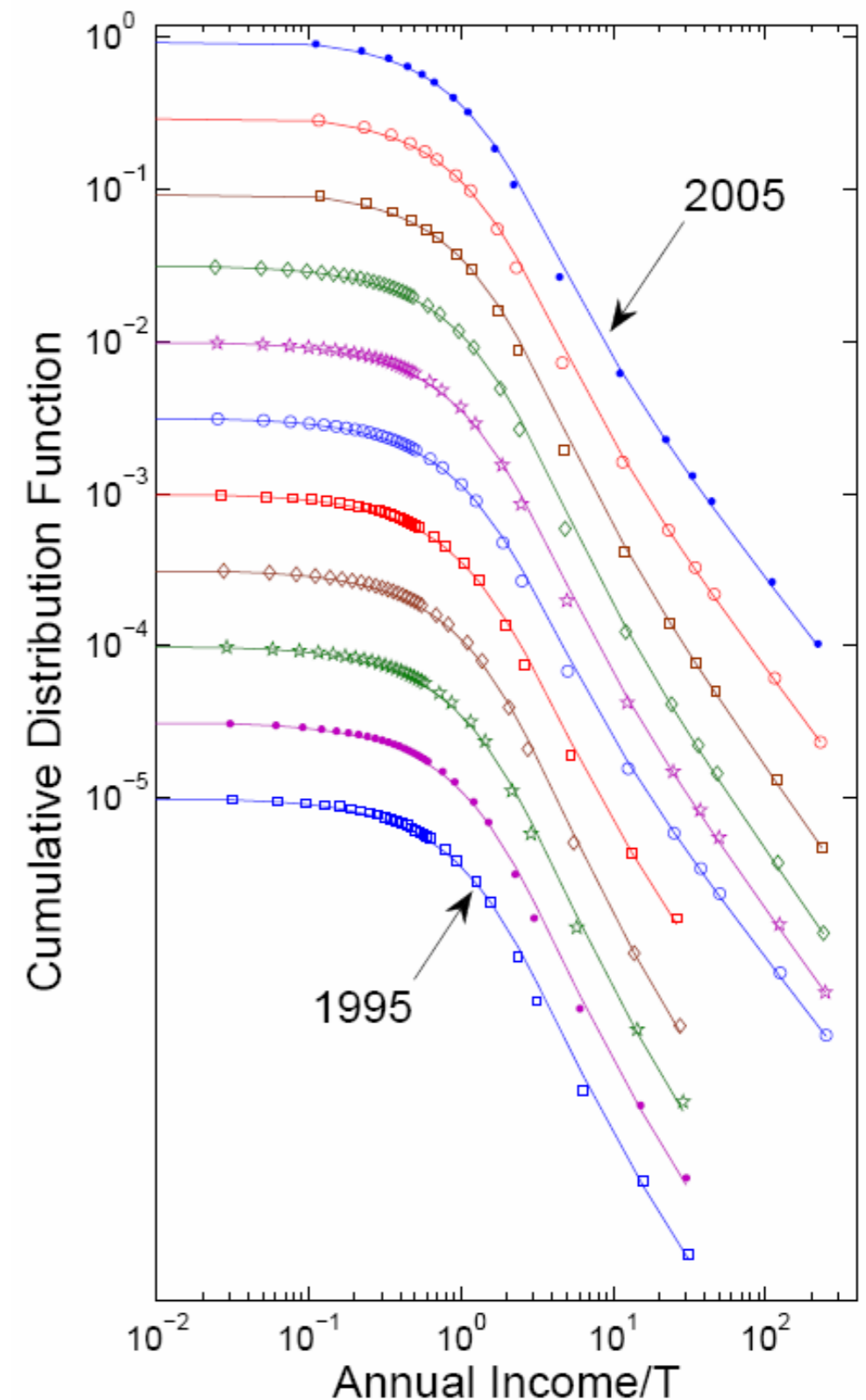
It interpolates between the exponential and the power-law distributions and has 3 parameters:

- $T = B_0/A_0$ – **temperature** of the exponential part
- $\alpha = 1 + a/b$ – **power-law exponent** of the upper tail
- r_0 – **crossover income** between the lower and upper parts.

Banerjee & Yakovenko, *RMP* (2009), *NJP* (2010)

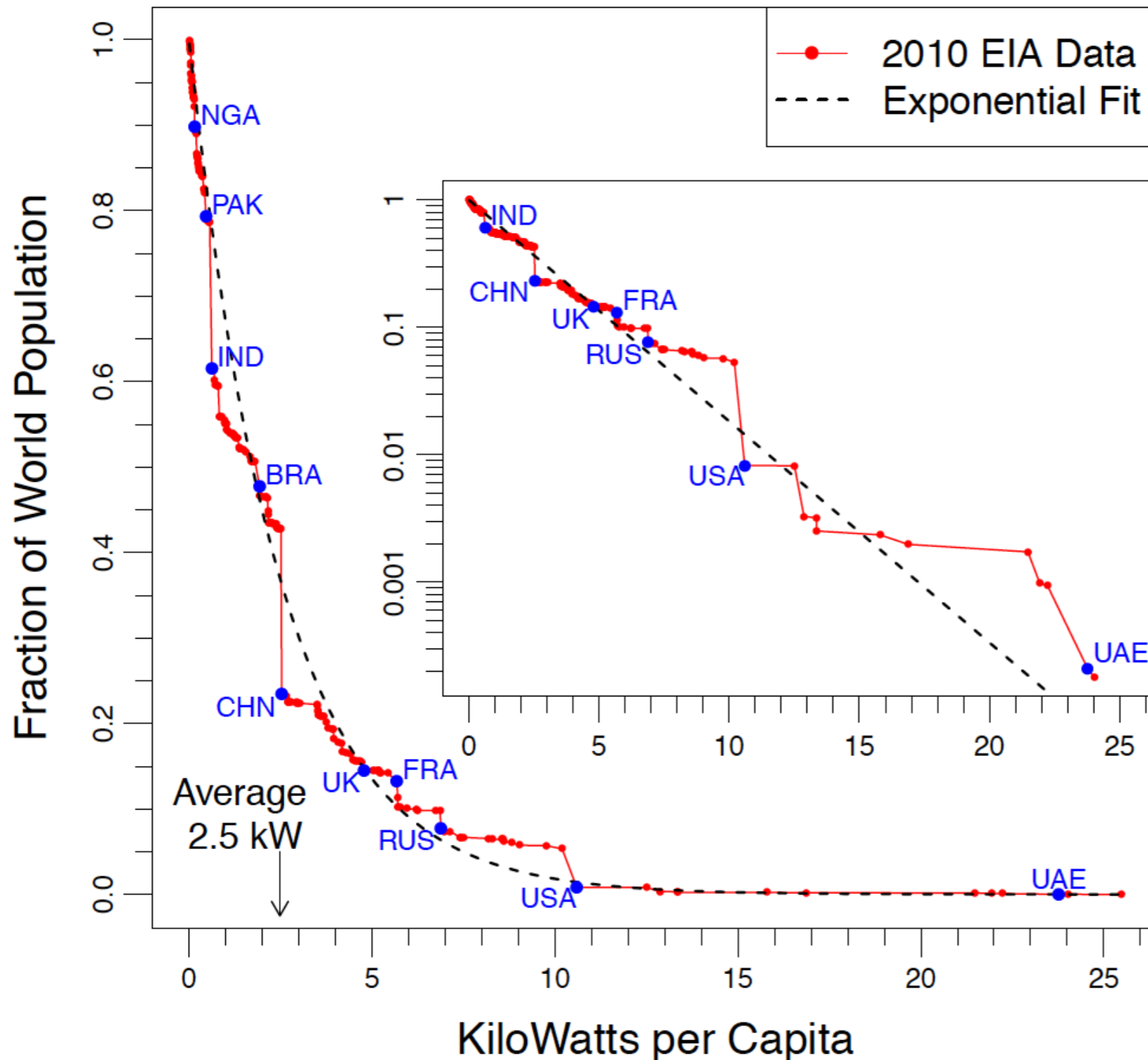
Fiaschi & Marsili, *JEBO* (2012)

Karl Pearson, *Proc. Roy. Soc. London* (1895)



Global inequality in energy consumption

Cumulative Distribution of Energy Consumption



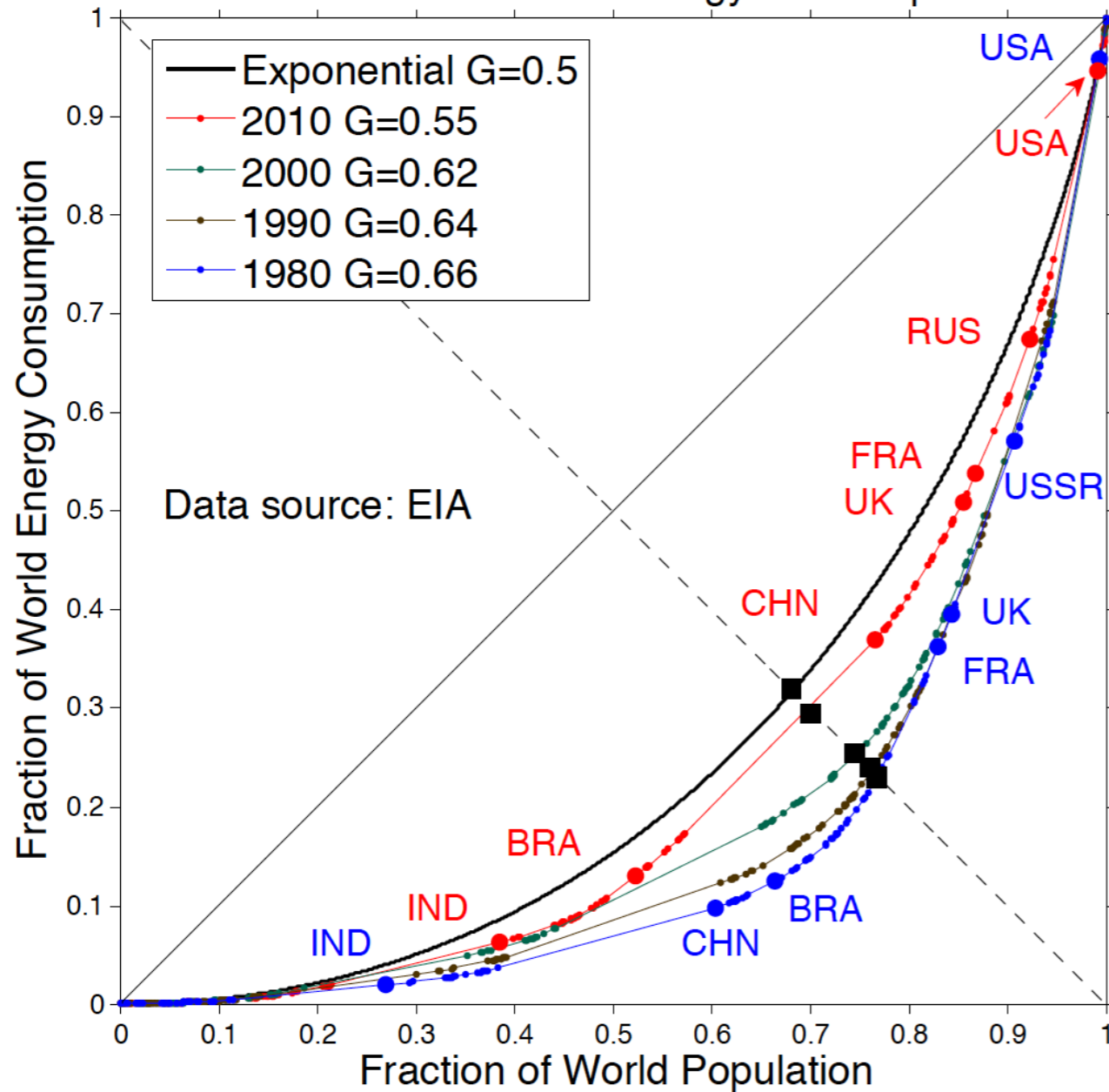
Global distribution of energy consumption per person is roughly **exponential**.

Division of a **limited resource + entropy maximization** produce **exponential distribution**.

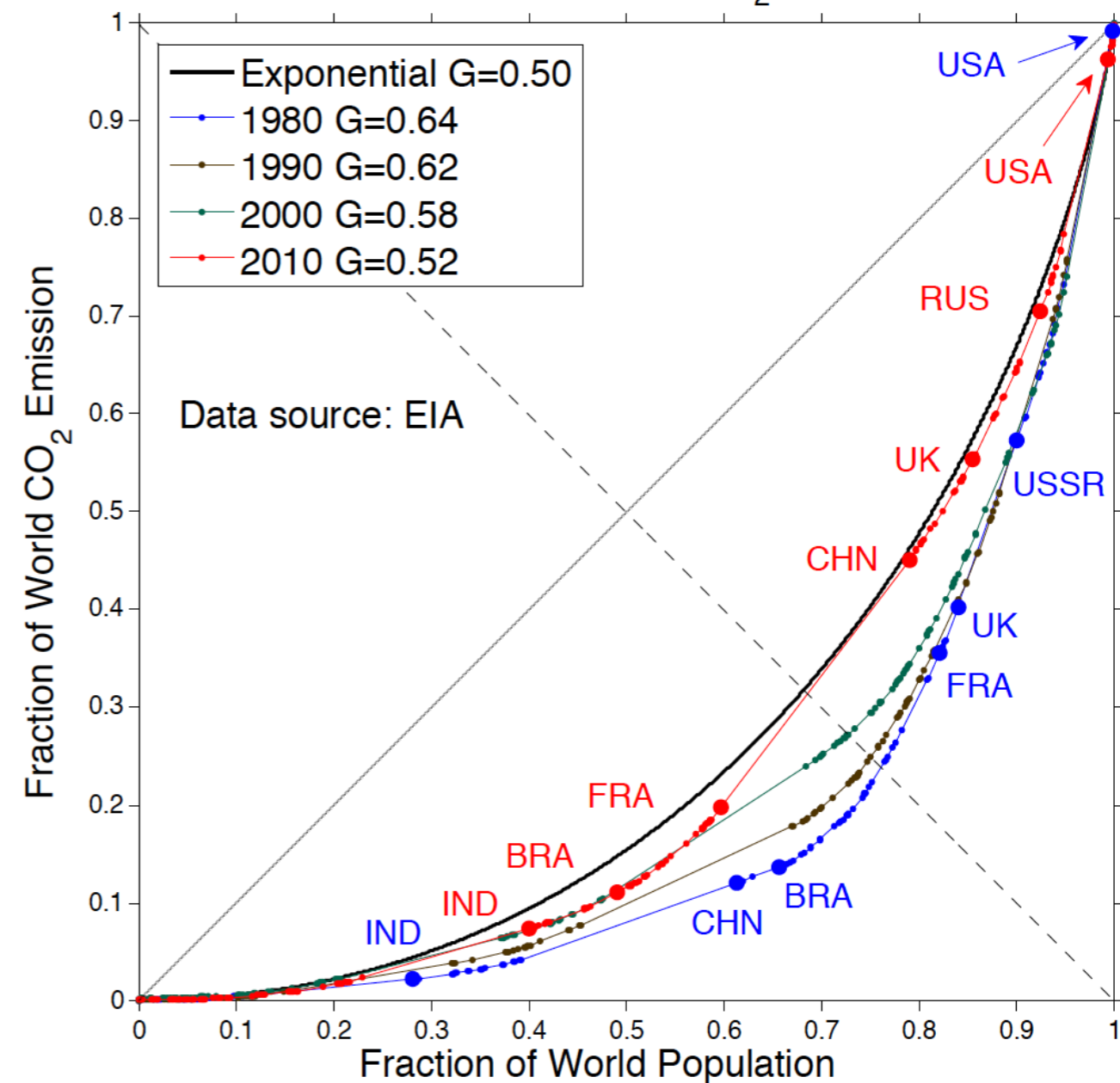
Physiological energy consumption of a human at rest is about **100 W**

Global inequality in energy consumption

Lorenz Plot of Global Energy Consumption

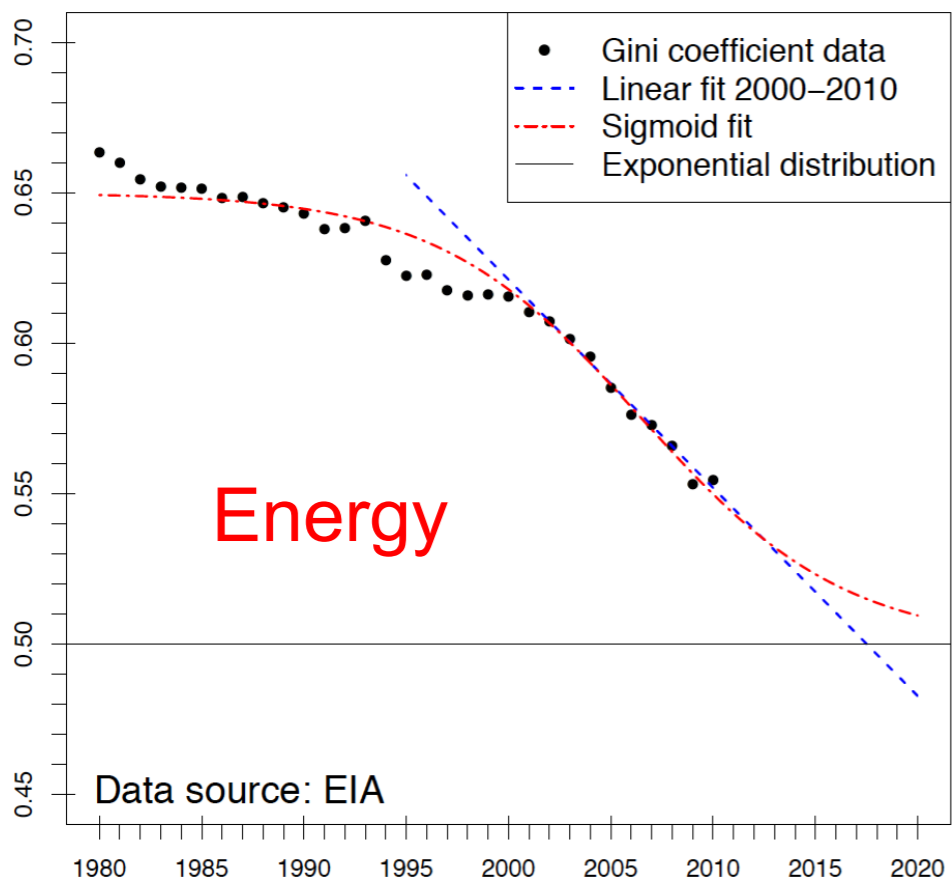


Lorenz Plot of Global CO₂ Emission

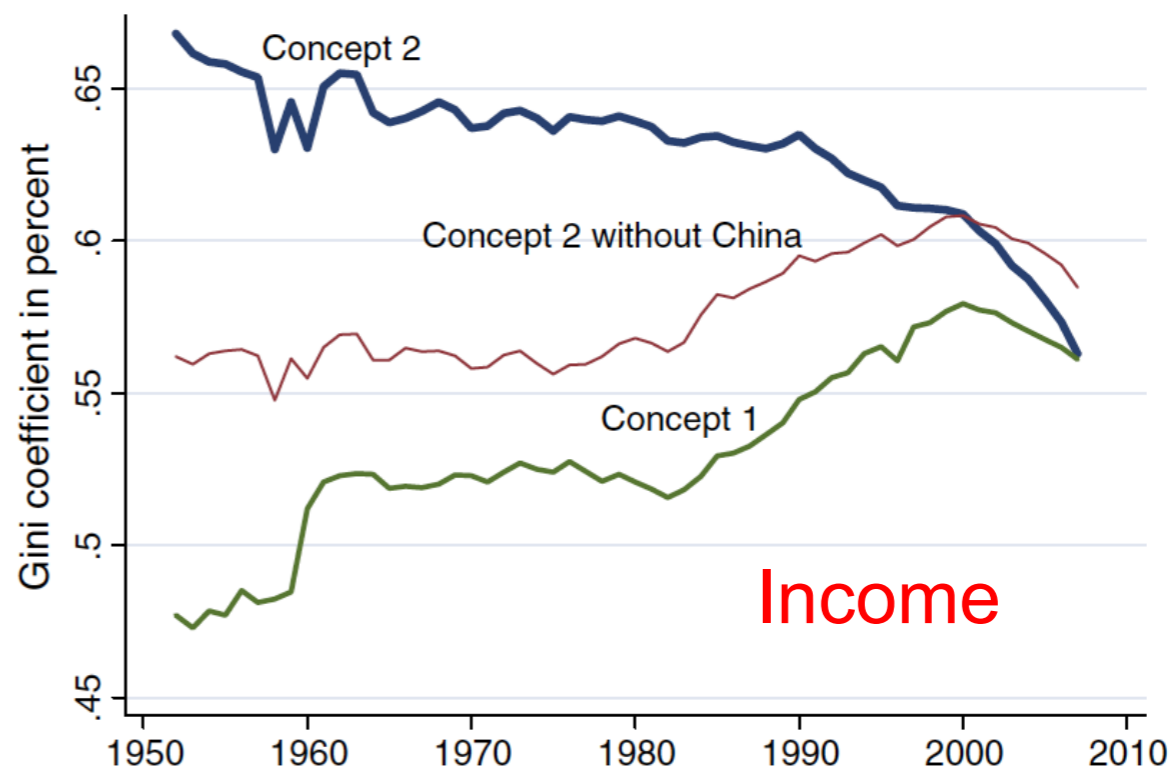


- Global **inequality** in energy consumption **decreases**.
- Energy consumption evolves toward the **exponential** distribution.
- **The law of 1/3**: Top **1/3** of world **population** consumes **2/3** of **energy**

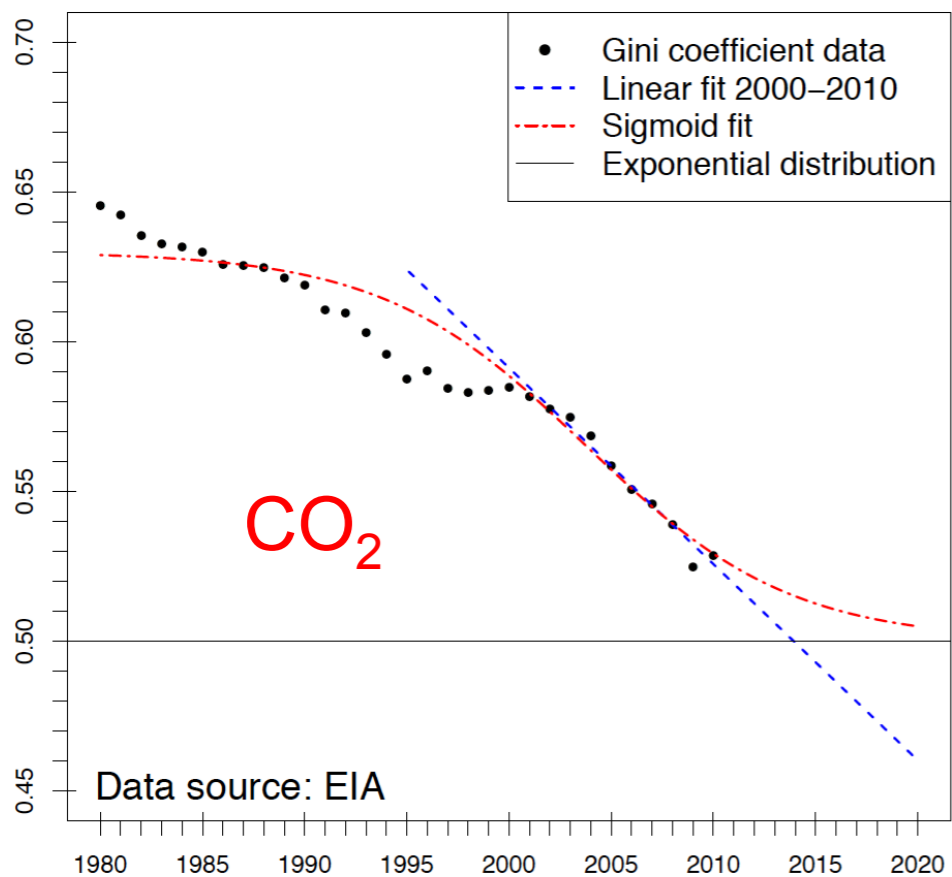
Gini Coefficient of Global Energy Consumption



B. Milanovic, *J Econ Inequal* 10, 1 (2012)

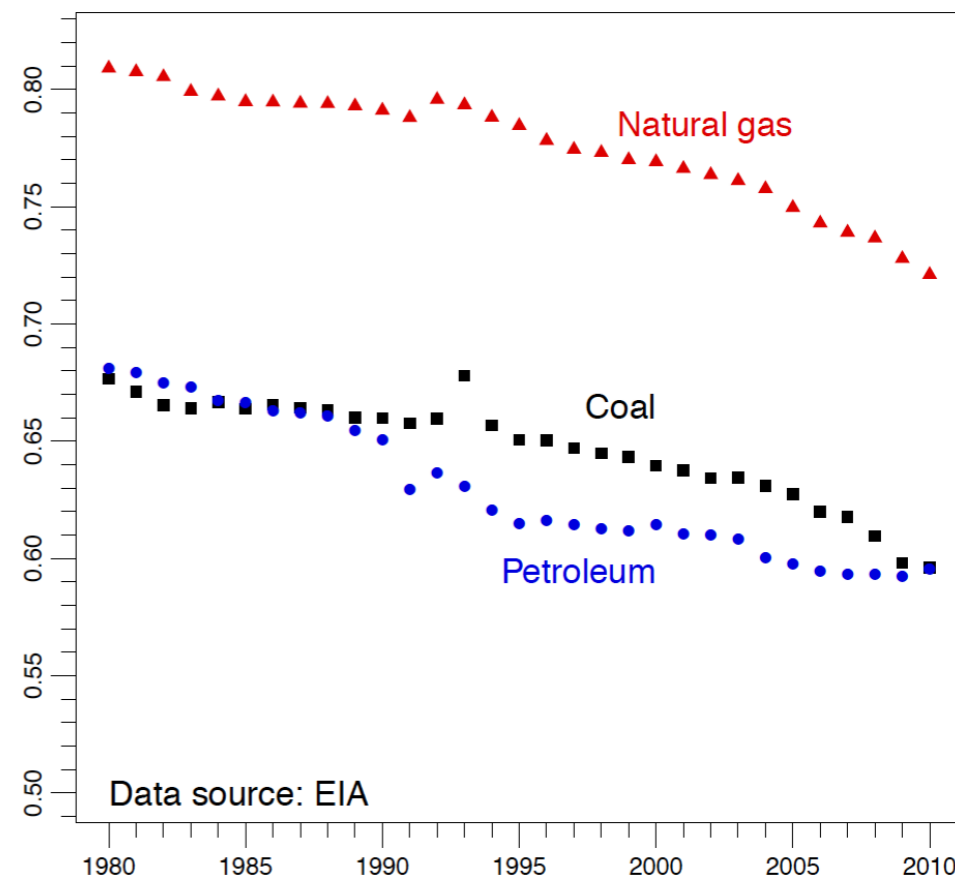


Gini Coefficient of Global CO₂ Emission



Money
 =
 Energy
 =
 Carbon

Gini Coefficients of Energy Consumption by Source



Conclusions

- The probability **distribution of money** is **stable** and has an **equilibrium** only when a **boundary condition**, such as $m > 0$, is imposed.
- When **debt** is permitted, the distribution of money becomes **unstable**, unless some sort of a **limit on maximal debt** is imposed.
- **Income distribution** in the USA has a **two-class structure**: **exponential** (“thermal”) for the great **majority (97-99%) of population** and **power-law** (“superthermal”) for the **top 1-3% of population**.
- The **exponential part** of the distribution is **very stable** and does not change in time, except for a **slow increase of temperature T** (the average income).
- The **power-law tail** is **not universal** and was increasing significantly for the last 20 years. It peaked and crashed in **2000** and **2007** with the **speculative bubbles** in financial markets.
- The global distribution of **energy consumption** per person is **highly unequal** and **roughly exponential**. This inequality is important in dealing with the global energy problems.
- All papers at <http://physics.umd.edu/~yakovenk/econophysics/>

Time evolution of income temperature

