Stable Swarming Using Adaptive Long-range Interactions

Dan Gorbonos

Based on:
- DG, Reuven Ianconescu, James Puckett, Rui Ni, Nicholas T. Ouellette and Nir S. Gov, NJP, 2016
- DG and Nir S. Gov, in preparation

Transport phenomena in collective dynamics: from micro to social hydrodynamics
03/11/16 ETH Zürich

WEIZMANN INSTITUTE OF SCIENCE

CHEMICAL PHYSICS Department
Outline

• Motivation
  - The midges and the swarm
  - The adaptive gravity model

• Stable Swarming with Adaptivity
  - Adaptivity as a self-stabilization mechanism
  - Jeans Instability

• Conclusions
The midges and the swarm

Chironomid Midge Fly Swarm
(Small White Spots in Image)
The midges and the swarm

In the lab (Stanford U.):

Nick Ouellette - PI
James Puckett
Rui Ni
The midges (Chironomidae)

- Non-biting midges
- Only male swarm (mating ritual)

<table>
<thead>
<tr>
<th>nature</th>
<th>lab</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many ?</td>
<td>10–10⁴</td>
</tr>
<tr>
<td>Where ?</td>
<td>stream edges</td>
</tr>
<tr>
<td>When ?</td>
<td>dawn and dusk</td>
</tr>
<tr>
<td></td>
<td>Black felt “swarm markers”</td>
</tr>
<tr>
<td></td>
<td>Overhead light source – ON/OFF</td>
</tr>
</tbody>
</table>
Method:

- High-speed stereo-imaging using three synchronized cameras (100 fps)
- Automated motion tracking algorithm

Measurement:

Kinematics –

\( \ddot{r}(t), \ddot{v}(t), \ddot{a}(t) \)
In the lab (Stanford U.):

- Long-range Interaction ("force")
- Swarm in the dark
- Not influenced by chemical signals
Isotropic Harmonic Oscillator

$$\sum \vec{F} = -K\vec{r}$$

Linear restoring force
– effective spring constant

Assumptions:
• Long range interaction
• Pairwise interaction
• uniform density
• spherical symmetry

The only possible force:
$$F \propto \frac{1}{r^2}$$

$$\sum \vec{F} \propto \int \frac{d^3r}{r^2} \hat{r} \propto \hat{r}$$
The Adaptive Gravity Model

The Model

- Acoustic attraction – Johnston’s organ
- Flight sound intensity decays as $\frac{1}{r^2}$

$\Rightarrow$ Acceleration towards the source $a \sim \frac{1}{r^2}$

- “Acoustic Gravity”

\[
\vec{F}_{eff} = C \sum_j \hat{r}_{ij} \frac{1}{|\vec{r}_i - \vec{r}_j|^2}
\]
Another feature (in the lab):

The linear force decreases for larger swarms

What is missing?
Adaptivity (as a part of the Fold Change Detection Mechanism)

- A typical feature of sensory systems

The Adaptive Gravity Model

**Scalar symmetry**

For any two stimuli $\vec{S}$ and $p \cdot \vec{S}$ ($p > 0$) the output is the same

\[ \vec{F}_{eff} \]

\[ \vec{F}_{eff} \left( p \cdot \vec{s}_{11}, ..., p \cdot \vec{s}_{ij}, ... \right) = \vec{F}_{eff} \left( \vec{s}_{11}, ..., \vec{s}_{ij}, ... \right) \]

Sensitive to directionality but not to the overall amplitude!
The Adaptive Gravity Model

Isotopic Harmonic Oscillator

with adaptivity

\[ F_{\text{eff}}^i = C \frac{\sum_j \hat{r}_{ij}}{\sum_j \left| \vec{r}_i - \vec{r}_j \right|} \]

Uniform density & spherical symmetry

\[ \sum_j \frac{1}{\left| \vec{r}_i - \vec{r}_j \right|^2} \rightarrow \int \frac{d^3r}{\left| \vec{r} - \vec{r}_s \right|^2} \sim R_s \]

\[ \vec{F} = K \vec{r} \]

effective spring constant

\[ K \sim \frac{1}{R_s} \]
Adaptive Gravity – Evidence

Supported by data – 122 swarms!

Black – raw data
Red – Binned average
Blue – (-1) slope (spherical) / (-2) slope (cylindrical)

Large swarms are elongated along the vertical axis
Dependence of The Effective Force on The Density (Uniform)

Regular gravity

\[ \frac{1}{r^2} \]

\[ \vec{s}_{ij} = C \frac{\hat{r}_{ij}}{r_{ij}^2} \]

\[ \vec{s}_i = \sum_j \vec{s}_{ij} \]

At the center:

\[ S(r) = -\frac{4\pi C \rho}{3} r \]

Adaptive gravity

\[ \frac{1}{r^2} \]

\[ \vec{F}_{eff}^i = \sum_j \vec{s}_{ij} \]

At the center:

\[ F_{eff}(r) = -\frac{\tilde{C}}{3R_s} r + O\left(\frac{r^3}{R_s^2}\right) \]

Adaptive forces

\[ \frac{1}{r^n} \quad (n > 2) \]

\[ \vec{s}_{ij} = C \frac{\hat{r}_{ij}}{r_{ij}^n} \]

\[ \vec{F}_{eff}^i = \sum_j \vec{s}_{ij} \]

At the center:

For \( n = 3 \):

\[ F_{eff}(r) = -\frac{2\tilde{C}}{R_s \ln(R_s^6 \rho^2)} r + O\left(\frac{r^3}{R_s^2}\right) \]

For \( n > 3 \):

\[ F_{eff}(r) = -\frac{\tilde{C}|n-3|}{3R_s (\rho \frac{3}{3} R_s^{n-3} - 1)} r + O\left(\frac{r^3}{R_s^2}\right) \]
Jeans Instability (Gravity)

- Balance: gravitational pull ⇔ random velocities
- \( \rho > \rho_{\text{Jeans}}^G \) ⇒ collapse (minimal density for collapse)
- If \( t_{\text{esc}} > t_{\text{col}} \)

\[
\begin{align*}
\text{Escape time (random velocities)} & \quad t_{\text{esc}} = \frac{R_s}{\sqrt{v^2}} \\
\text{Time for collapse} & \quad t_{\text{col}} = \frac{\pi}{2\sqrt{k}} \\
\text{Force} & \quad \vec{F} = -k\vec{r} \\
\text{For force} & \quad k = \frac{4\pi C}{3} \rho \\
\text{Jeans density} & \quad \rho_{\text{Jeans}}^G = \frac{3\pi \sqrt{v^2}}{16R_s^2 C}
\end{align*}
\]
Jeans Instability (Adaptive Gravity)

- No critical density $\rho_{\text{Jeans}}$

- If $t_{\text{esc}} > t_{\text{col}}$

  - Escape time (random velocities)
    \[ t_{\text{esc}} = \frac{R_s}{\sqrt{v^2}} \]

  - Time for collapse
    \[ t_{\text{col}} = \frac{\pi}{2\sqrt{k}} \]

  - Force
    \[ \vec{F} = -k\vec{r} \]

  - Condition
    \[ k = \frac{C}{3R_s} \Rightarrow \frac{4R_sC}{3\pi^2} > \frac{v^2}{R_s} \]
Jeans Instability (Adaptive Forces $\frac{1}{r^n} \ (n > 2)$)

- Stabilization at a particular density $\rho^A_{\text{Jeans}}$

- If $t_{\text{esc}} > t_{\text{col}}$
  
  Escape time (random velocities)
  
  \[ t_{\text{esc}} = \frac{R_s}{\sqrt{v^2}} \]

  Time for collapse
  
  \[ t_{\text{col}} = \frac{\pi}{2\sqrt{k}} \]

  \[ n = 3 \quad k = \frac{2\tilde{C}}{R_s \ln(R_s^6 \rho^2)} \quad \Rightarrow \quad \rho^A_{\text{Jeans}} = \frac{1}{R_s^3} \exp\left(\frac{4\tilde{C}R_s}{\pi^2 v^2}\right) \]

  \[ n > 3 \quad k = \frac{\tilde{C}|n - 3|}{3R_s(\rho^3 R_s^{n-3} - 1)} \quad \Rightarrow \quad \rho^A_{\text{Jeans}} = \frac{1}{R_s^3} \left(1 + \frac{4\tilde{C}|n - 3|R_s}{3\pi^2 v^2}\right)^{\frac{3}{n-3}} \]
Midge swarm dynamics is dominated by long range acoustic interactions.

The interactions are adaptive - weaker when the background intensity is higher.

Adaptivity, for general power-law interactions, stabilizes the swarm against collapse.

A prediction: A Selection of a particular density for higher power law interactions \((n > 2)\)
Acknowledgements

Experimental group (Yale U.):

Nir Gov – PI

Reuven Ianconescu (Research Assistant)

Nick Ouellette - PI

James Puckett

Rui Ni
THE END
Extended Virial Theorem

\[ 2T + W - \int p\vec{r} \cdot d\vec{s} = 0 \]

Surface Pressure – keeps the swarm together

Density profile

Poisson-Boltzmann equation w/ cut-off

Ellipsoidal Approximation

Boundary closer to the center – Stiffer effective spring