

# Modeling of Fronts and Patterns at the Atomic level for Surface Reactions

## CO oxidation on metals (100) surfaces

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KI-Net Conference

# Pattern Formation in Surface Reactions

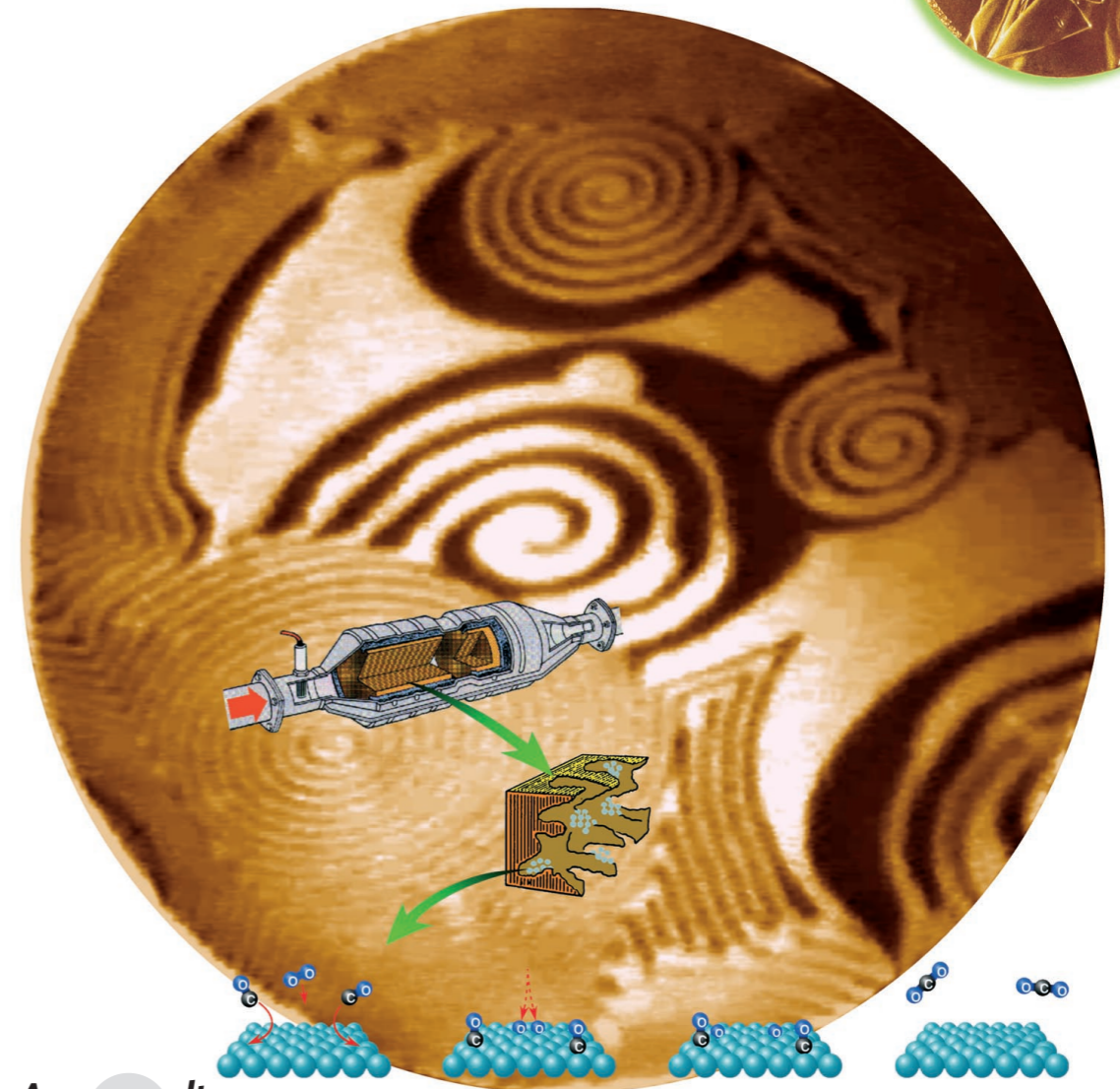
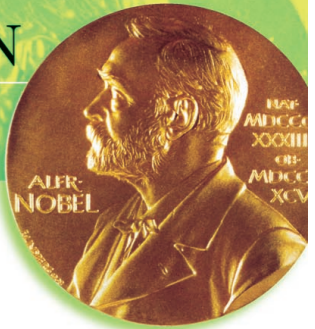
PEEM of CO oxidation on Pt(110)  
(Rotermund, Ertl 1993)



Reviews

G. Ertl

THE NOBEL PRIZE IN  
CHEMISTRY 2007



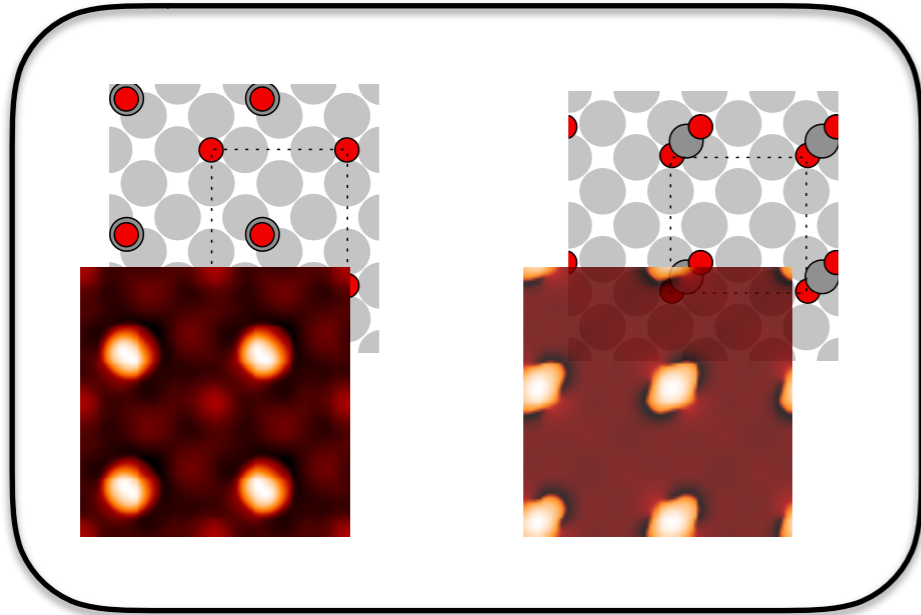
Angewandte  
Chemie

3524 www.angewandte.org

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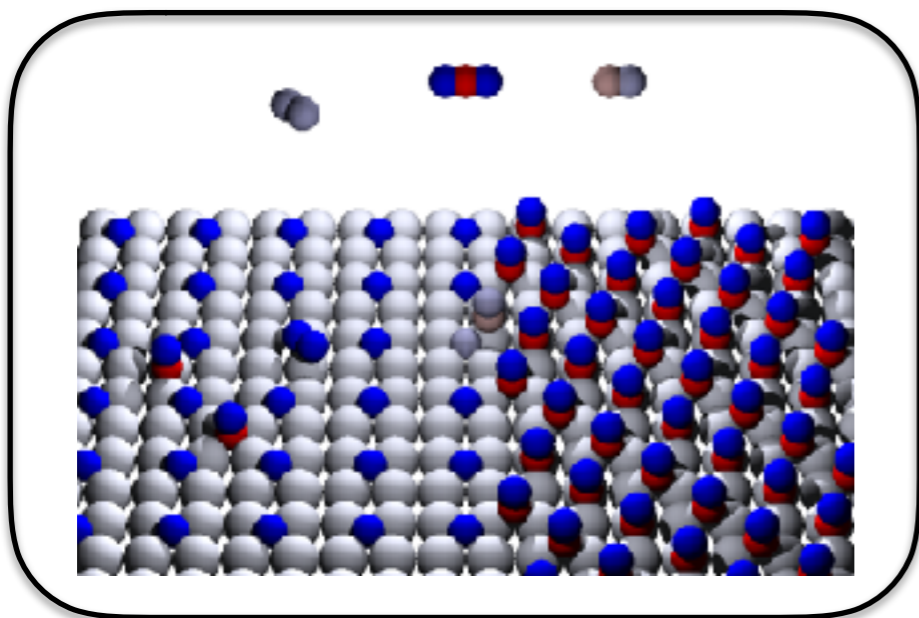
Angew. Chem. Int. Ed. 2008, 47, 3524–3535

Electronic level: DFT (PW and GTO)  $\rightarrow$  CCSD(T)  
 $\rightarrow$  QMC(?) (Zahariev, Gordon, Ames Lab)

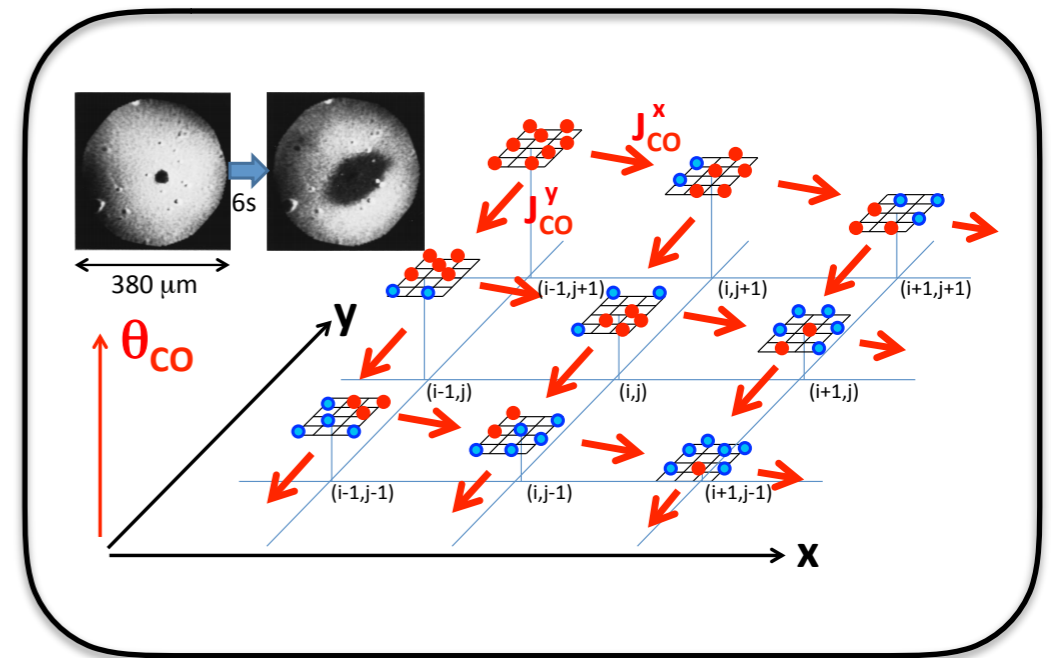
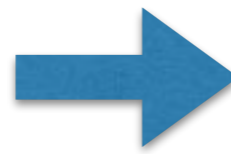


Phenom.  $\rightarrow$  Cluster Expansion  
 $\rightarrow$  machine learning(?)

Multi-site Lattice-Gas model

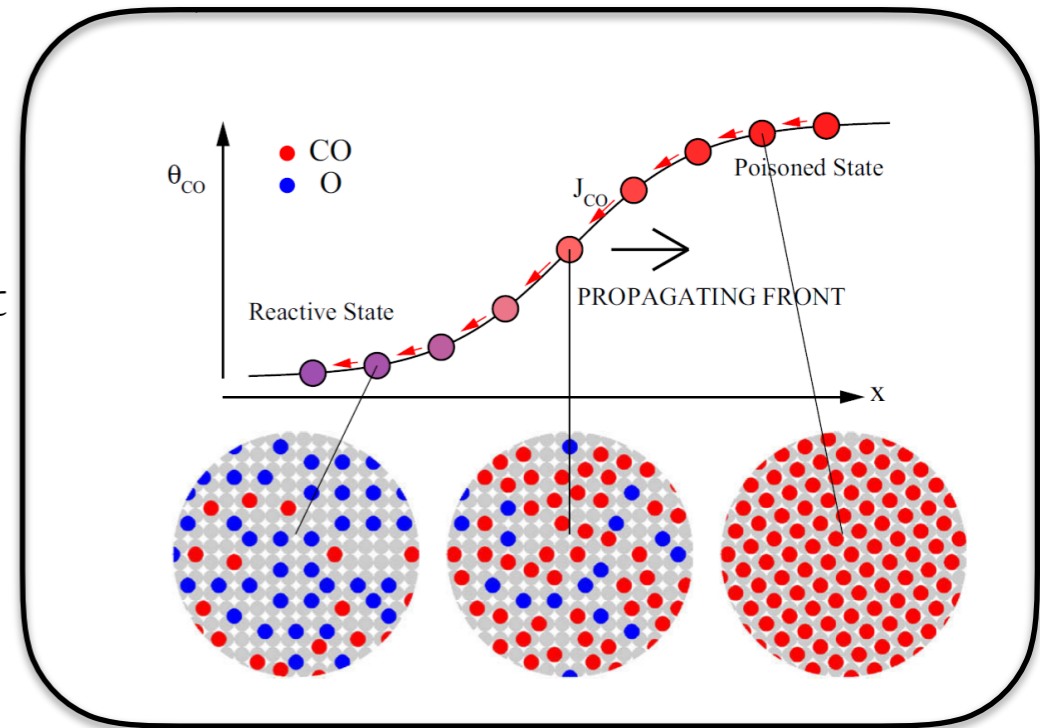


Multi-species transport  
 Onsager theory



massively parallel  
 kMC simulations  
 (Xeon-Phi, GPU)(?)

Heterogenous coupled  
 lattice-gas model (HCLG)



# Electronic Level

DFT (at the GGA level) is very good in determining interaction between species

- but not very good at site preference, diffusion barrier
- Inadequate for adsorption and reaction energies.

Chemical bond on metal surfaces: adsorbate interacts with whole surface rather than their nearest neighbors; single calculation using model system (periodic or cluster geometry) is not reliable

Averaging technique: varying slab thicknesses, for (100) surfaces,  $L=7$  to 12 layers.

# Multisite Lattice-Gas Model

For homogenous surfaces:

$$H = \sum_{i,\alpha} n_i^\alpha \epsilon_i^\alpha + \sum_{i < j} \sum_{\alpha,\beta} n_i^\alpha n_j^\beta \omega_{ij}^{\alpha,\beta}(d)$$

- $\alpha$  denotes various species at different adsorption sites, e.g. CO(br), O(h), ...
- $i$  and  $j$  denotes various lattice sites
- Pairwise interactions only
- $\omega$  determined directly from DFT (PBE);  $\epsilon$  from DFT corrected by CCSD(T) value of CO triplet-singlet gap.

### CO-CO interactions

substrate	$d/a$	top-top	br-br	4fh-4fh	mixed
Pd(100)	1	0.285	0.301	0.447	
	$\sqrt{5}/2$				0.141
	$\sqrt{2}$	0.000	0.006	0.065	
	$3/2$				0.005
	$\sqrt{5}/2$	-0.004	0.011	-0.004	
	2	0.019	0.020	0.020	
	$3/\sqrt{2}$		0.003		
Rh(100)	1	0.289	0.172	0.317	
	$\sqrt{5}/2$				0.109
	$\sqrt{2}$	-0.001	0.023	0.038	
	$\sqrt{5}/2$	0.009	0.009	0.009	
	2	0.016	0.027	0.024	
Pt(100)	1	0.314	0.273	0.454	
	$\sqrt{5}/2$				0.146
	$\sqrt{2}$	-0.012	0.017	0.059	
	$\sqrt{5}/2$	-0.002	0.024	-0.002	
Ir(100)	1	0.288	0.143	0.356	
	$\sqrt{5}/2$				0.084
	$\sqrt{2}$	-0.008	0.020	0.047	
	$\sqrt{5}/2$	0.006	0.004	0.006	
	2	0.005	0.012	0.009	

### CO-O interactions

substrate	$d/a$	top	br	4fh	mixed
Pd(100)	1	0.106	0.208	0.438	
	$\sqrt{5}/2$				0.172
	$\sqrt{2}$	-0.003	0.032	0.078	
	$3/2$				0.020
Rh(100)	$\sqrt{5}/2$	0.003	0.021	0.003	
	1	0.164	0.149	0.370	
	$\sqrt{5}/2$				0.154
	$\sqrt{2}$	-0.001	0.038	0.044	
Pt(100)	$3/2$				0.020
	$\sqrt{5}/2$	0.010	0.021	0.010	
	1	0.088	0.192	0.370	
Ir(100)	$\sqrt{5}/2$				0.159
	$\sqrt{2}$	-0.024	0.033	0.109	
Ir(100)	1	0.130	0.113	0.361	
	$\sqrt{5}/2$				0.129
	$\sqrt{2}$		0.023		

### O-O interactions

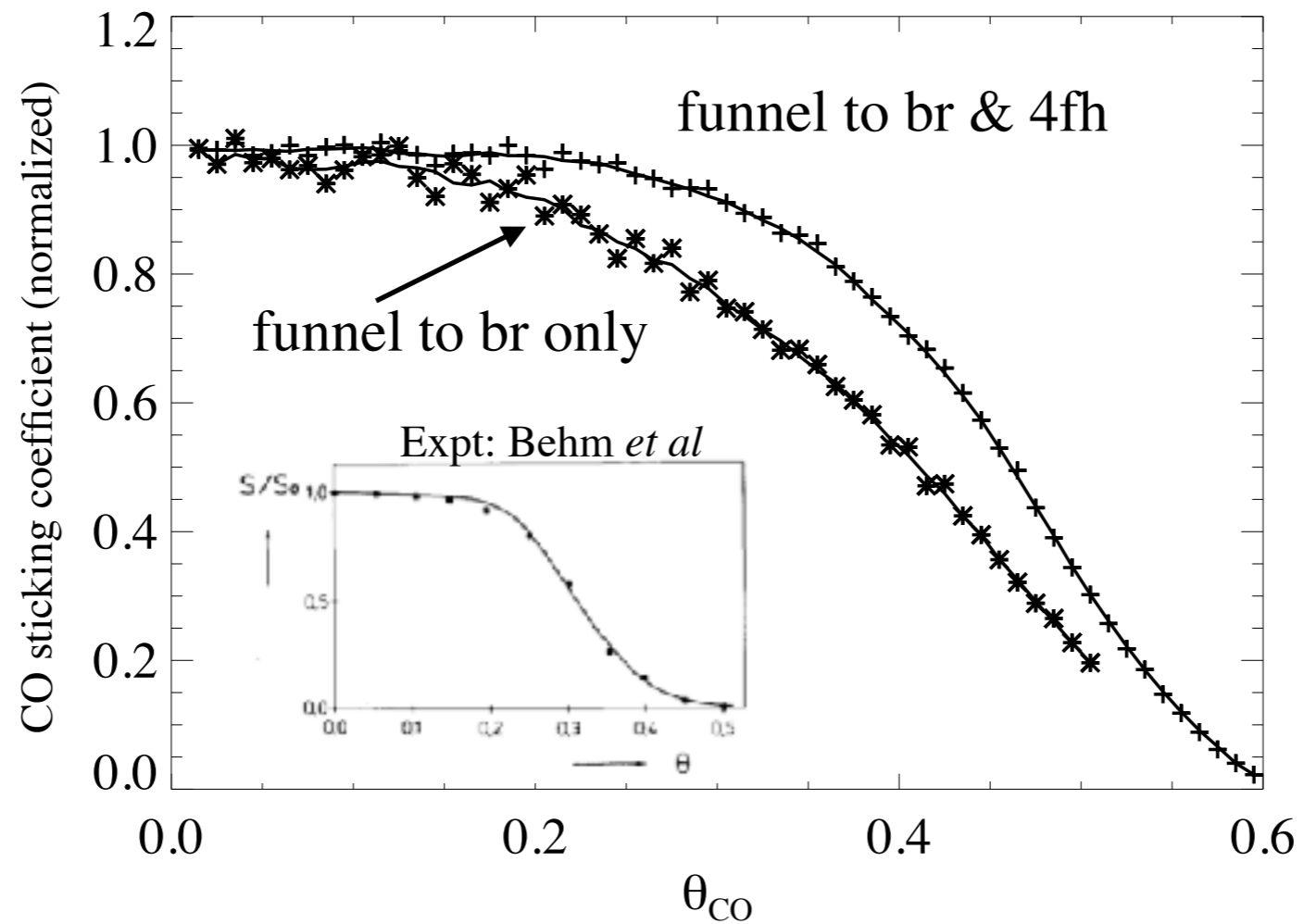
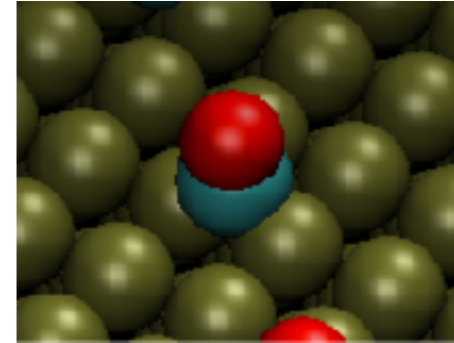
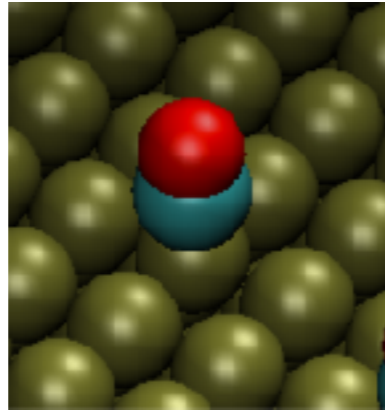
substrate	$d/a$	top	br	4fh	mixed
Pd(100)	1	0.142	0.219	0.368	
	$\sqrt{5}/2$				0.247
	$\sqrt{2}$	0.058	0.086	0.132	
	$3/2$				0.047
	$\sqrt{5}/2$	0.017	0.054	0.017	
	2		0.013	-0.036	
Rh(100)	$\sqrt{5}$			-0.002	
	1	0.148	0.238	0.464	
	$\sqrt{5}/2$				0.214
	$\sqrt{2}$	0.028	0.056	0.141	
	$\sqrt{5}/2$	0.013	0.037	0.014	
Pt(100)	2		0.012	-0.005	
	$\sqrt{5}/2$			0.001	
	1	0.045	0.208	0.339	
	$\sqrt{5}/2$		-0.107(v), 0.341(g)		0.188
	$\sqrt{2}$	-0.016	0.014	0.180	
Ir(100)	$\sqrt{5}/2$	0.003	0.015	0.003	
	2		0.060	0.003	
	$\sqrt{5}/2$		0.071(v), 0.017(g)		
	1	0.050	0.162	0.378	
	$\sqrt{5}/2$		-0.077(v), 0.390(g)		0.194
Ir(100)	$\sqrt{2}$	-0.008	0.010	0.124	
	$\sqrt{5}/2$	-0.002	0.044	-0.002	
	2		0.025	0.005	
	$\sqrt{5}/2$		0.030(v), -0.020(g)		
			-0.005		

### adsorption energy

M	Pd	Rh	Pt	Ir
br CO	1.644	1.708	1.854	1.741
4fh CO	1.426	1.420	1.154	1.216
top CO	1.344	1.726	1.904	2.033
br O	0.940	1.840	1.231	1.960
4fh O	1.220	2.020	0.719	1.641
top O	-0.220	0.085	0.100	1.184

# CO Adsorption Kinetics:

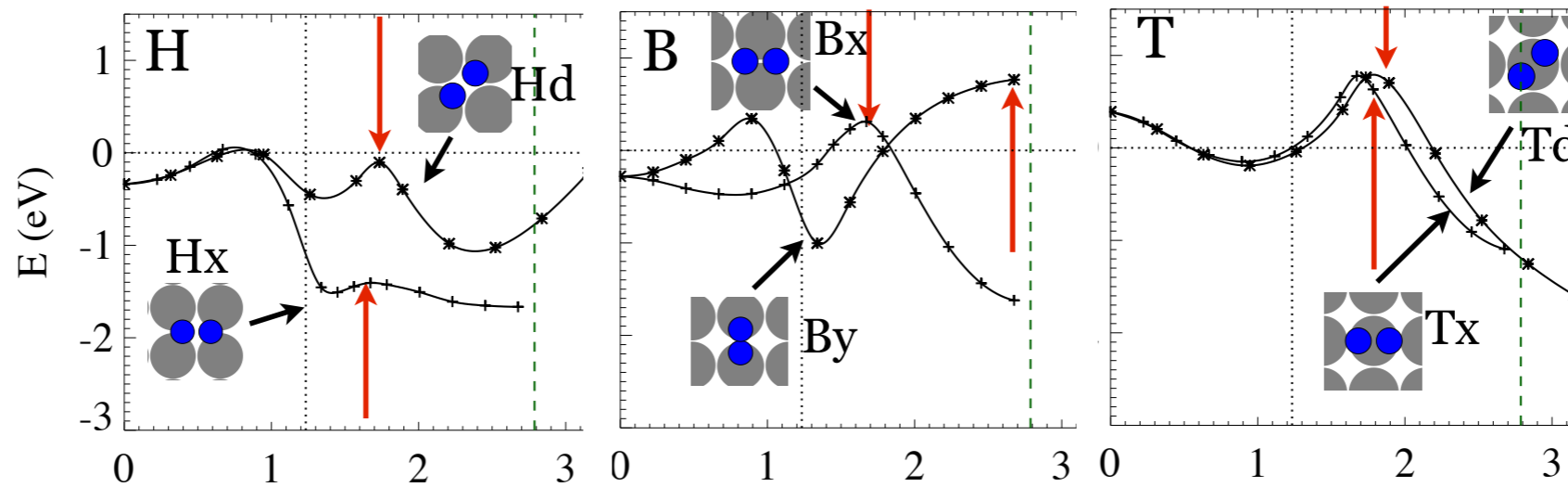
CO steered to top site first, then funnel to a nearby bridge site



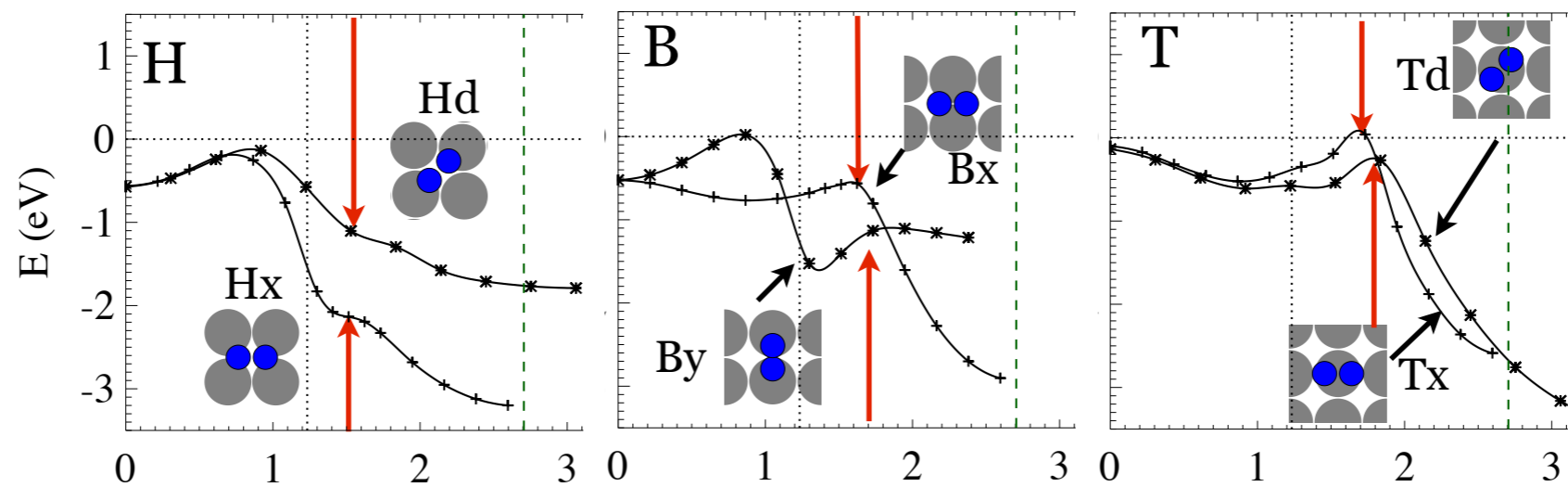
# O<sub>2</sub> Dissociation Kinetics:

Constrained minimal energy path from DFT

(a) O<sub>2</sub>/Pd(100)



(b) O<sub>2</sub>/Rh(100)



Distance between O projected to the surface (Å)

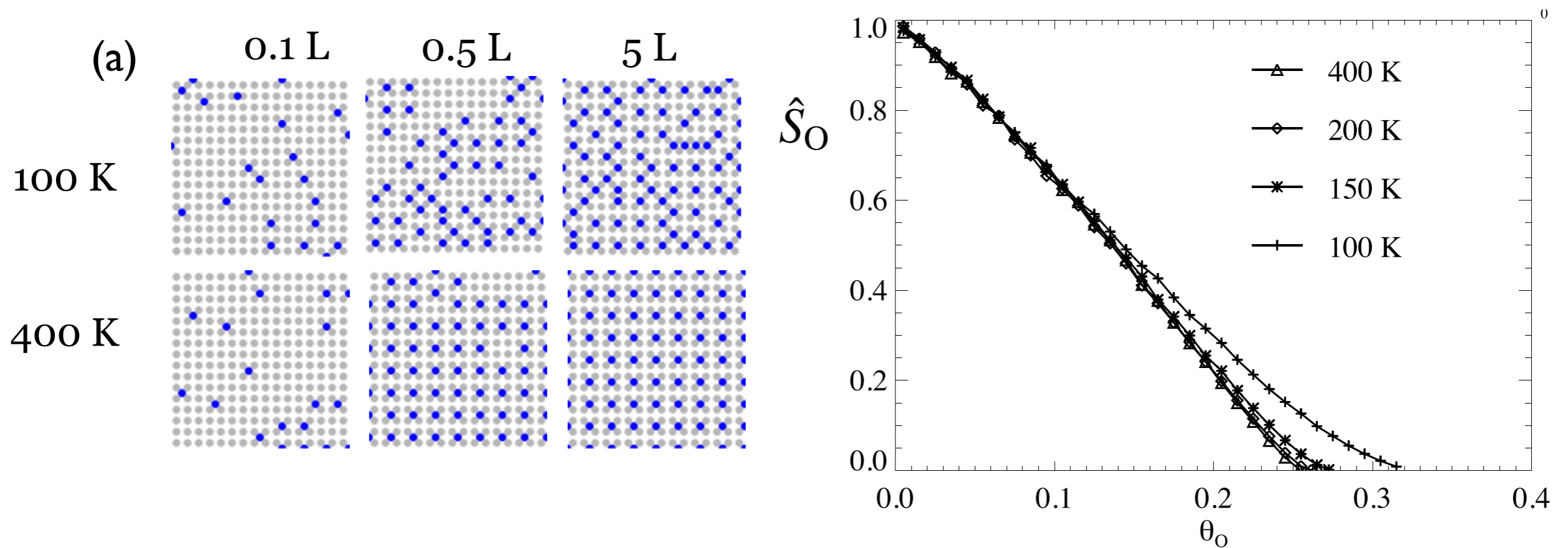


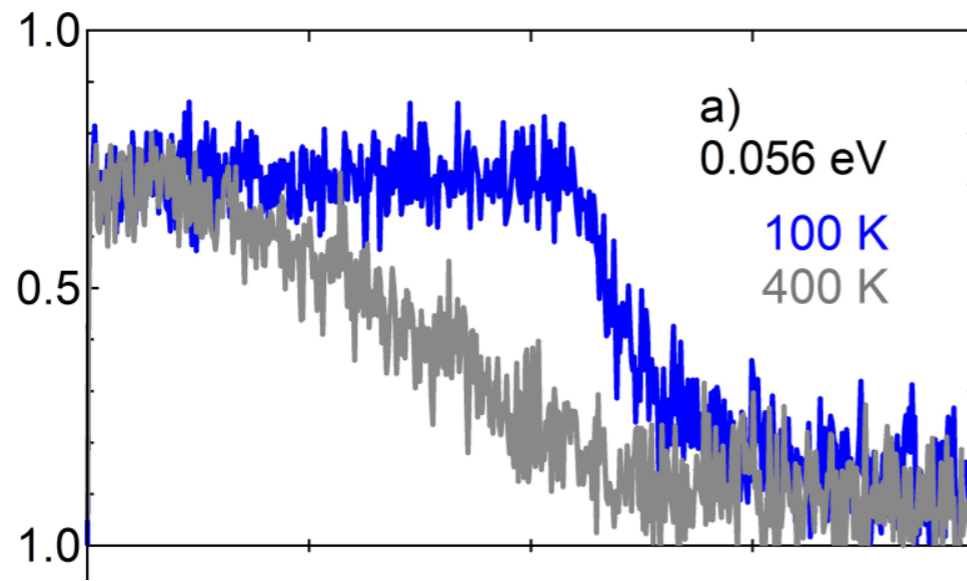
# A General LG Model for O<sub>2</sub> Dissociation

$$E_{\text{act}} = E_{\text{ads}} + \Delta E_{\text{TS}},$$

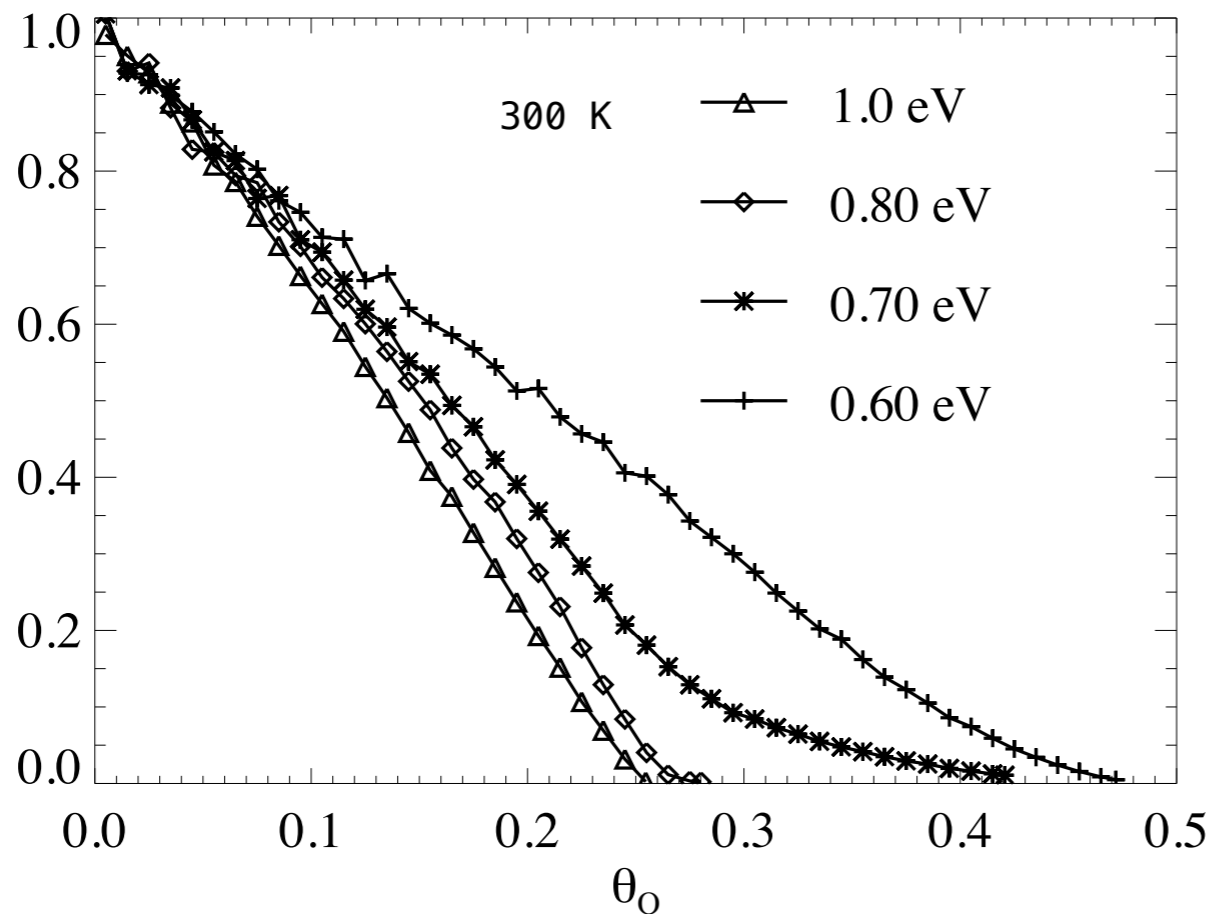
$E_{\text{ads}}$ : Adsorption energy of 2 O(ads) as NN, including lateral interactions

$\Delta E_{\text{TS}}$ : Extra barrier to the transition state energy for them to truly dissociate, not sensitive to environment, only as function of the pathway.

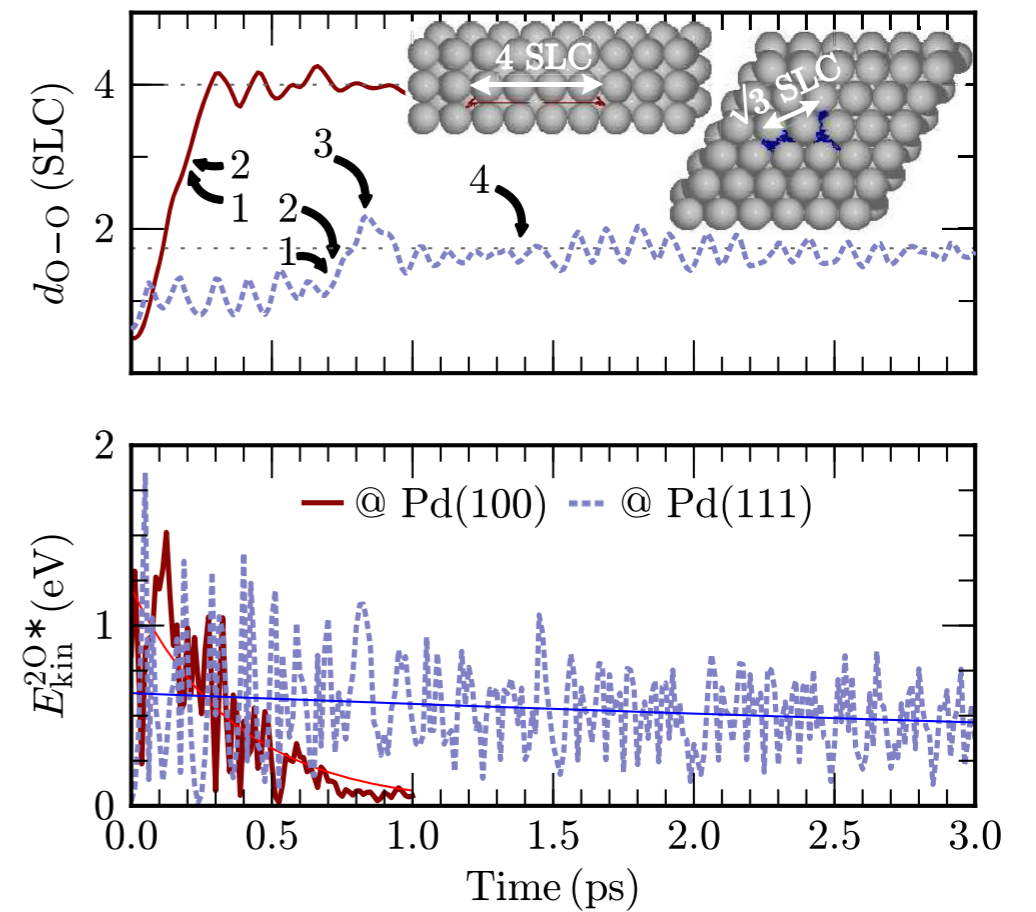




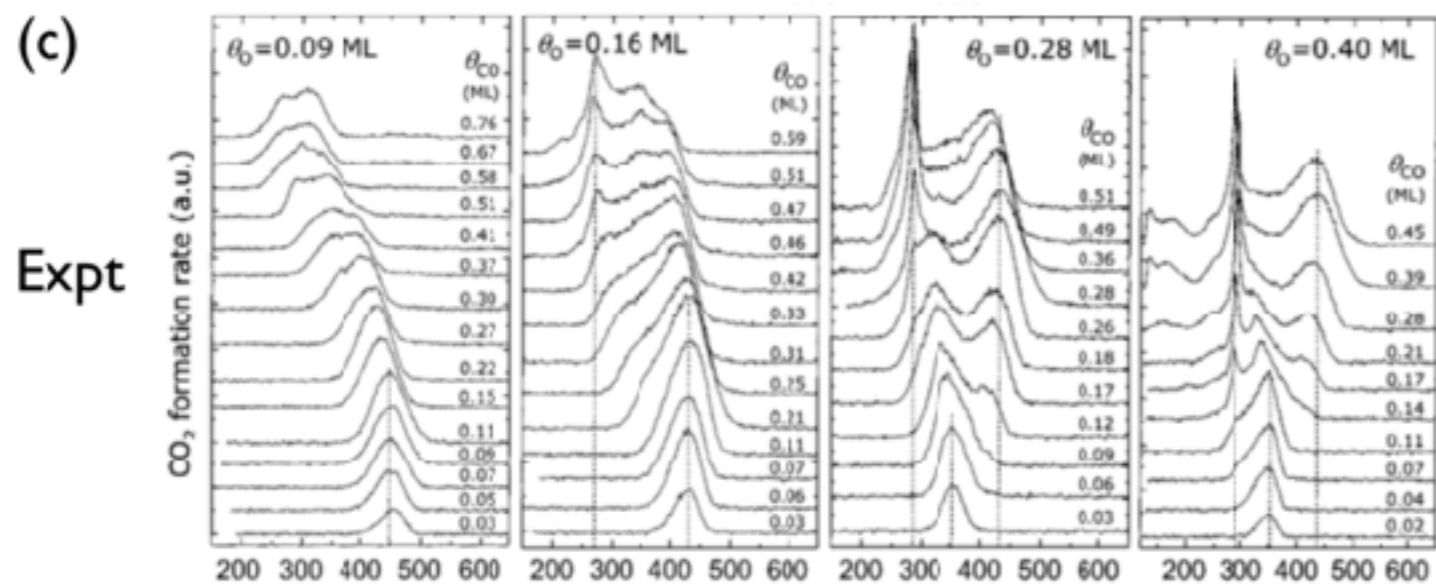
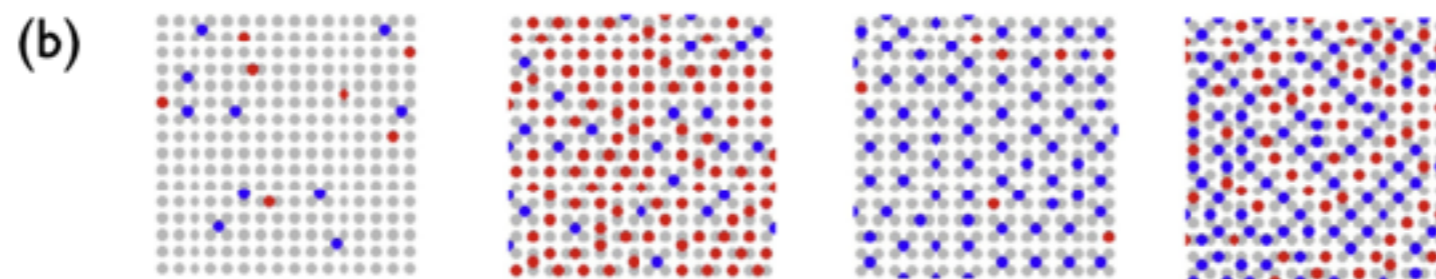
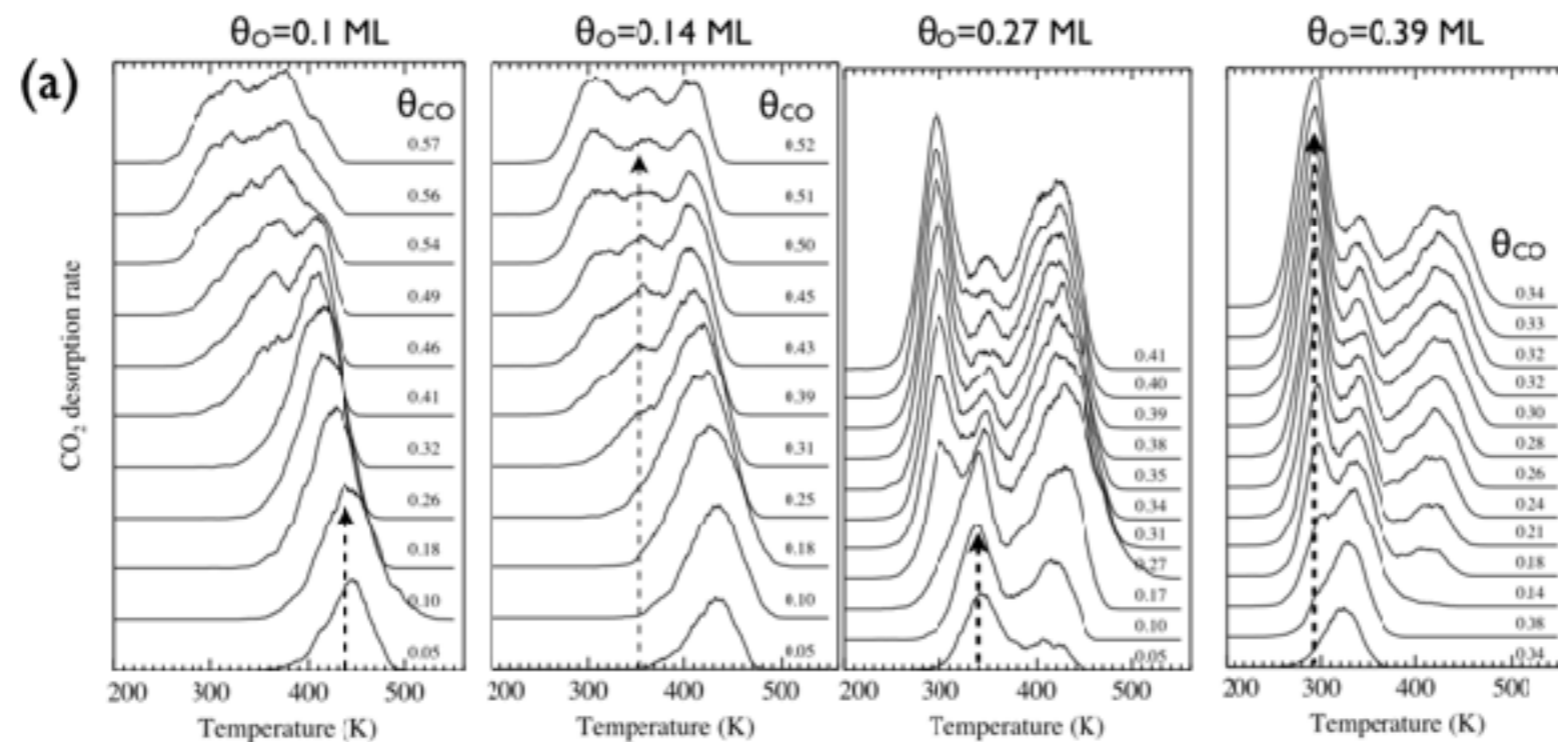
Normalized O<sub>2</sub> Sticking on Pd(100)  
Refined model with different  $\Delta E_{TS}$  for Hx pathway



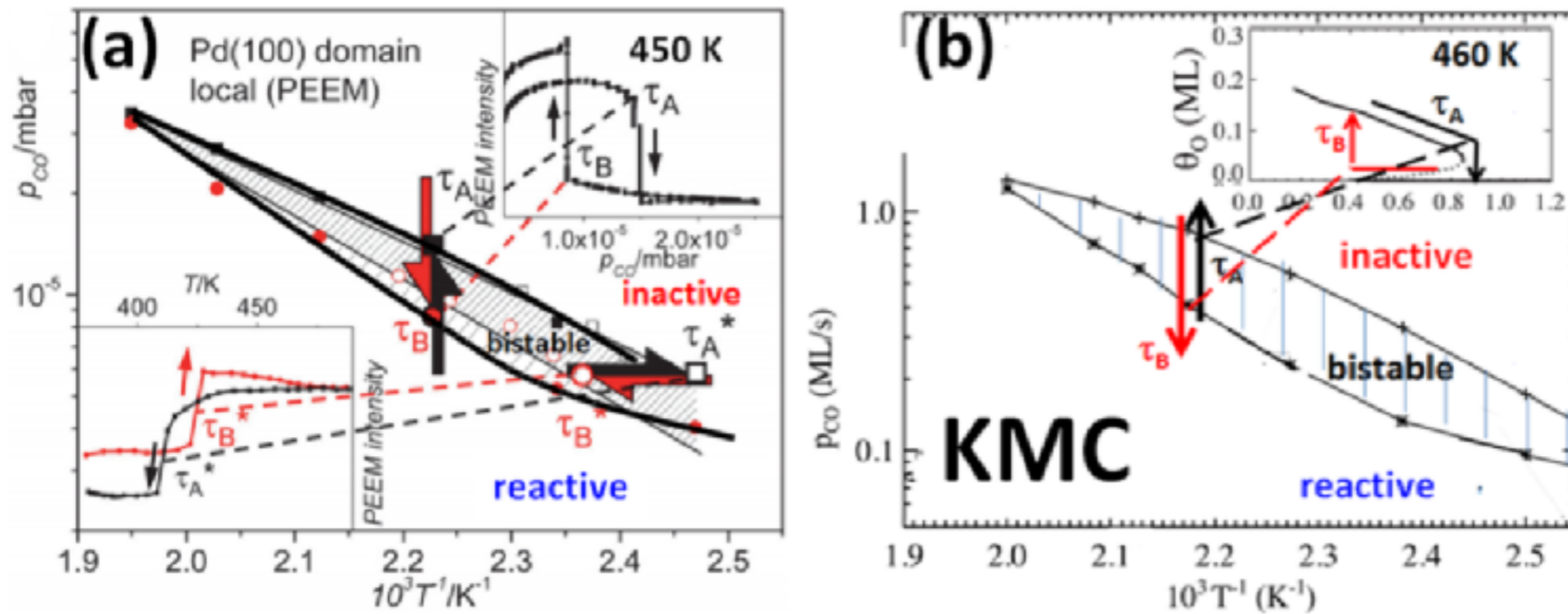
Hot adatom diffusion  
Bukas and Reuter, PRL **117**, 146101 (2016)



# Temperature-Programmed Reaction Spectra CO+O/Rh(100)



# Bifurcation Diagram *CO Oxidation on Pd(100)*



Vogel *et al*

# Tailored Model for CO Oxidation on Pd(100)

CO on bridge sites only; O on 4fh sites only

No neighboring pairs with  $d \leq a$ .

No interaction between reactants with  $d > a$ .

Dissociative adsorption of O<sub>2</sub> satisfying the above prescription of interaction.

CO desorption with activation barrier  $E_{CO}$

CO+O reaction at 2NN 4fh-bri pair with  $E_{react}$

Very fast CO diffusion, moderately fast O diffusion

Almost as simple, but more physical than the widely studied Ziff-Gulari-Barshad (ZGB) model for CO oxidation

Only two parameters needed, unfortunately DFT not reliable for neither of them.

# Kirkwood-type Approx. of the Tailored Model

O<sub>2</sub> dissociation

9 fh + 8 bri blocking, reorientation

$$S_{\text{CO}} = [1 - 2\theta_{\text{CO}} - 2\theta_{\text{O}}][1 - \alpha\theta_{\text{CO}}^4] / [1 - 2\theta_{\text{CO}} - 2\theta_{\text{O}} + 9.5\theta_{\text{CO}}^3 + \theta_{\text{CO}}^2\theta_{\text{O}} + 2\theta_{\text{O}}^6]$$

$$S_{\text{O}_2} = 2P_{9+8} - P_{13+16}, \quad \text{where} \quad P_{9+8} = Q_{8|9}P_9$$

$$\text{and} \quad P_{13+16} = Q_{16|13}P_{13}$$

Kirkwood approximations:

$$P_9 = P_2^8 / P_1^8$$

$$P_{13} = P_2^{16} / P_1^{19}$$

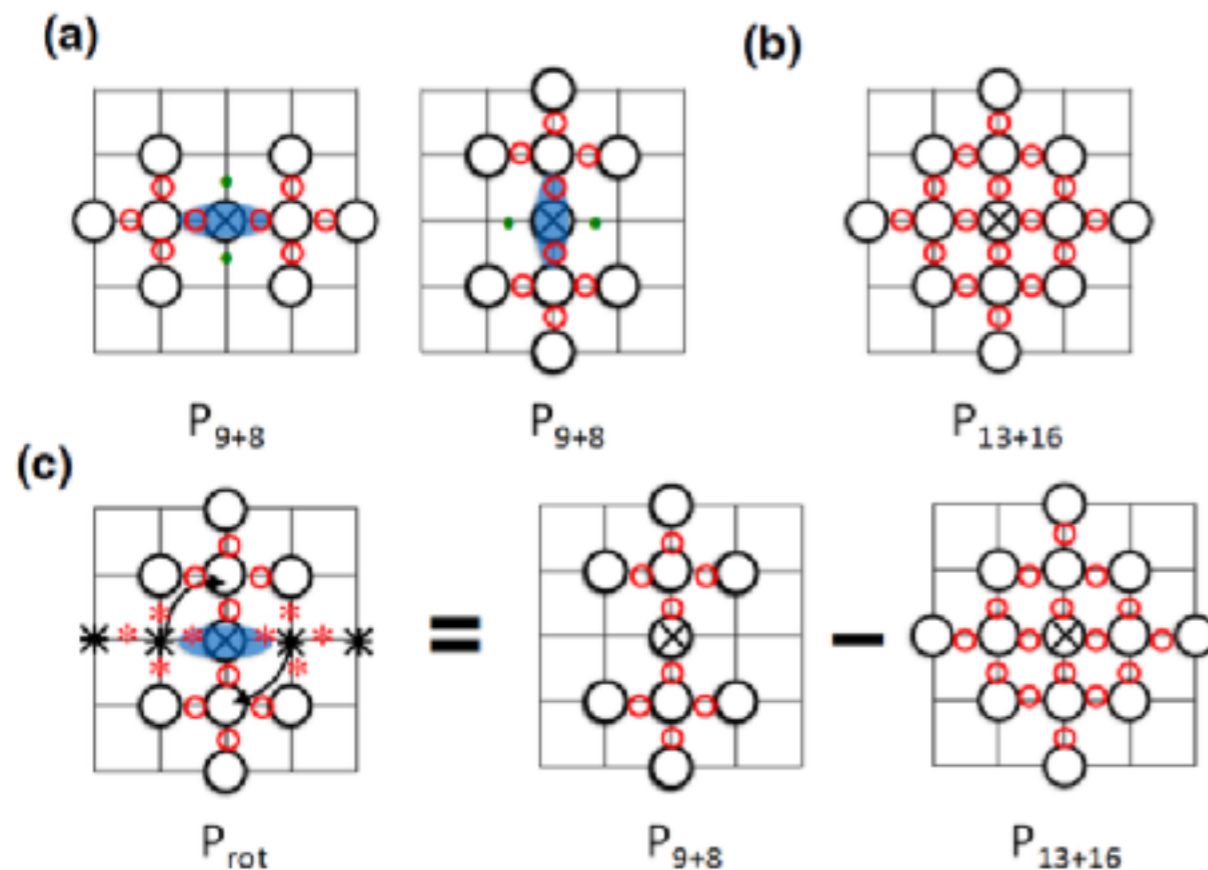
$$P_1 = 1 - \theta_{\text{O}}$$

$$P_2 = 1 - 2\theta_{\text{O}}$$

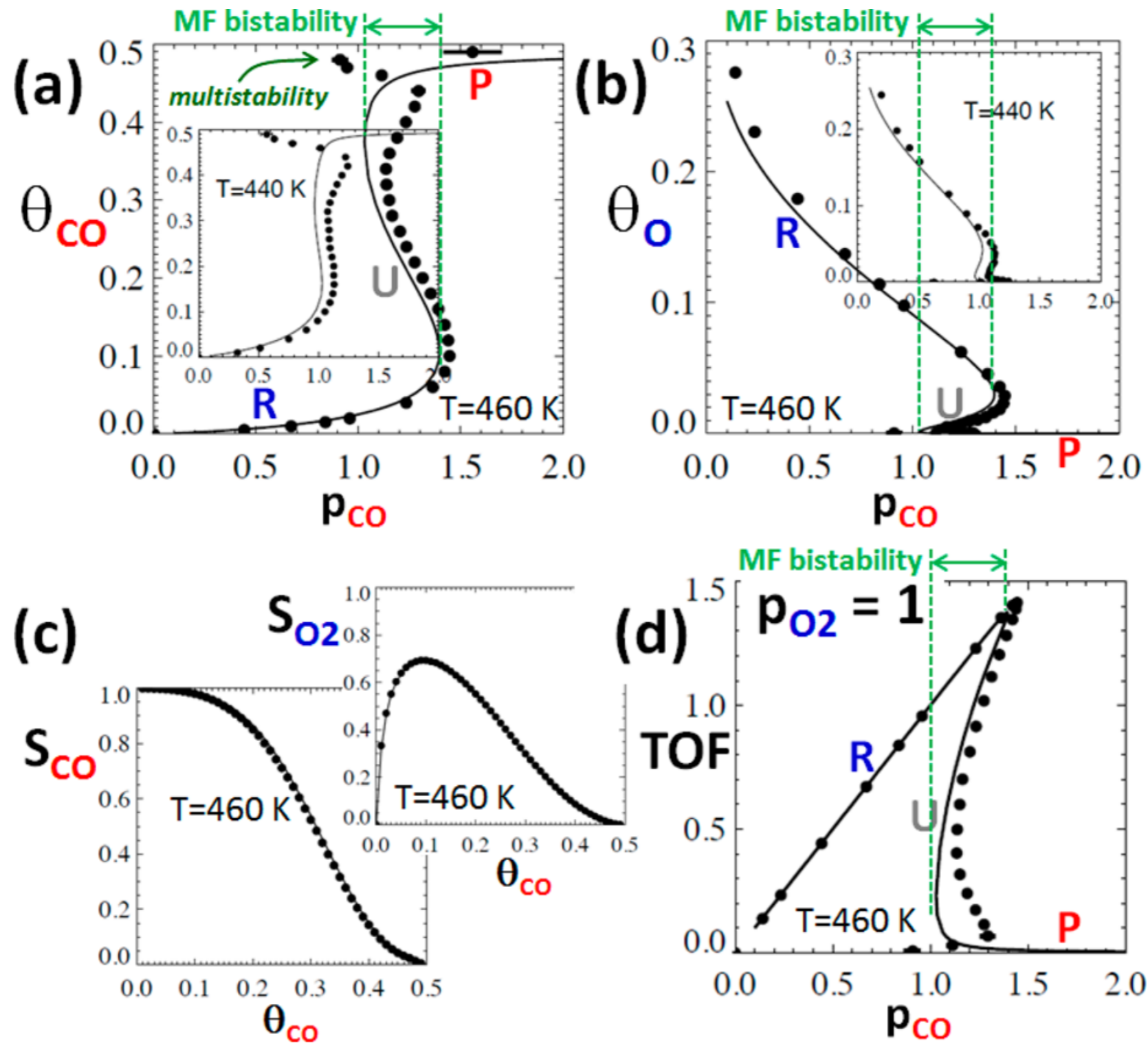
$$Q_{8|9} = Q_4^2 Q_2 / Q_1^2$$

$$Q_n = 1 - n\theta_{\text{CO}}^{\text{br}}(\text{loc})$$

$$\theta_{\text{CO}}^{\text{br}}(\text{loc}) = \theta_{\text{CO}}^{\text{br}} / (1 - 2\theta_{\text{O}})$$

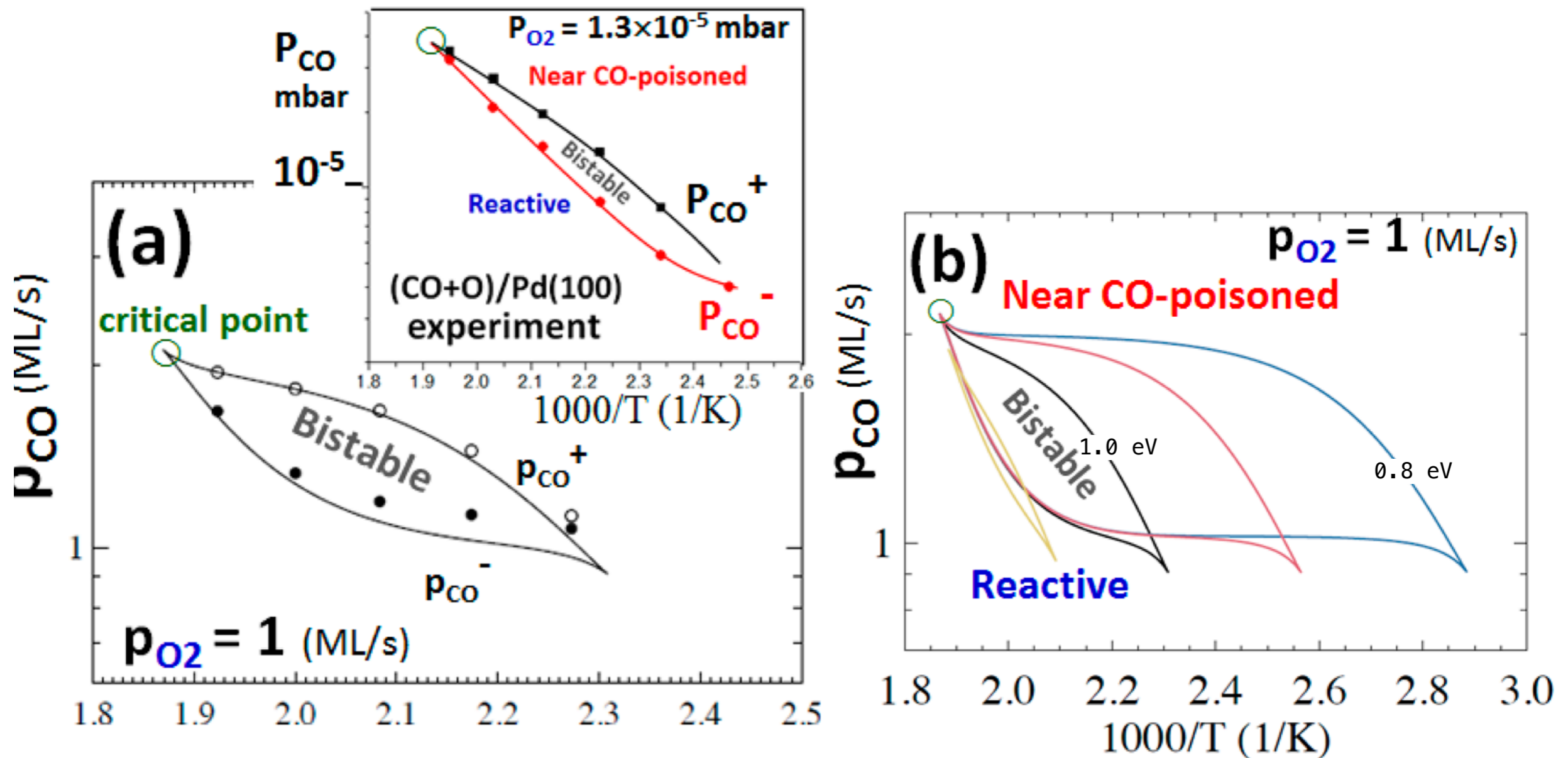


# Bistability in the Tailored Model



# Phase diagram of the Tailored Model

## Numerical Continuation (Homotopy) Method





# Reaction-Diffusion Equation of Mixed Reactants

Revised Fick's Laws of Diffusion

$$\frac{\partial \theta_{\text{CO}}}{\partial t} = R_{\text{CO}}(\theta_{\text{CO}}, \theta_{\text{O}}) - \nabla \cdot \mathbf{J}_{\text{CO}},$$

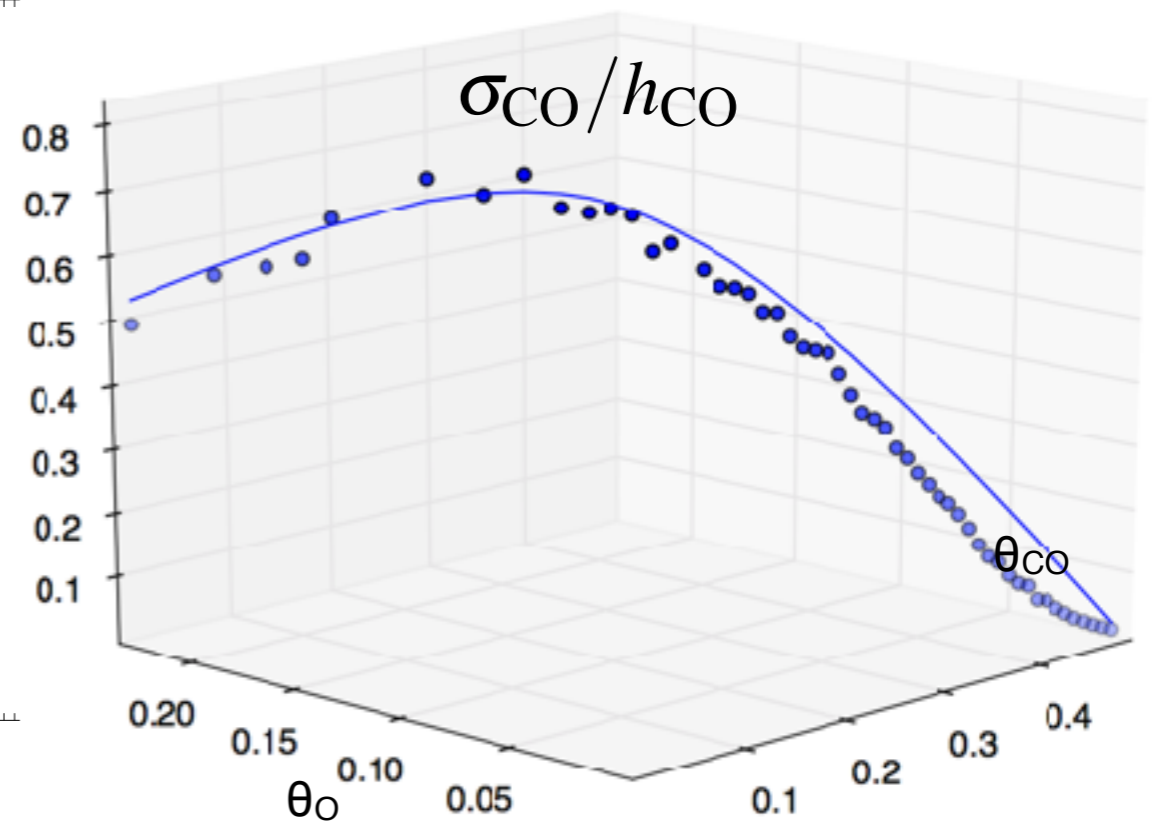
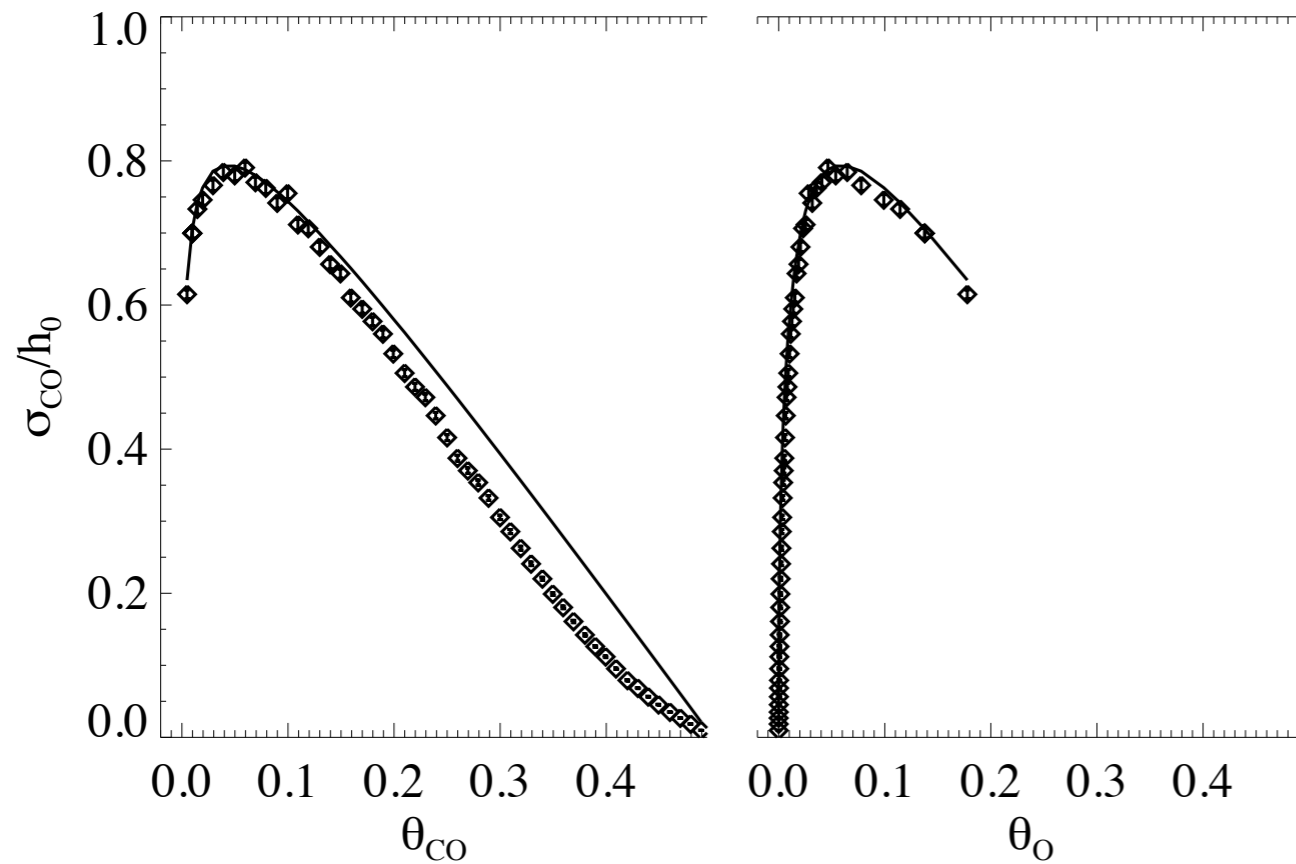
$$\frac{\partial \theta_{\text{O}}}{\partial t} = R_{\text{O}}(\theta_{\text{CO}}, \theta_{\text{O}})$$

$$\mathbf{J}_{\text{CO}} = -D_{\text{CO,CO}} \nabla \theta_{\text{CO}} - D_{\text{CO,O}} \nabla \theta_{\text{O}}$$

Onsager theory:

$$J_{\text{CO}} = -\Lambda_{\text{CO}} \nabla \mu_{\text{CO}}$$

# The mobility of CO in mixed layers



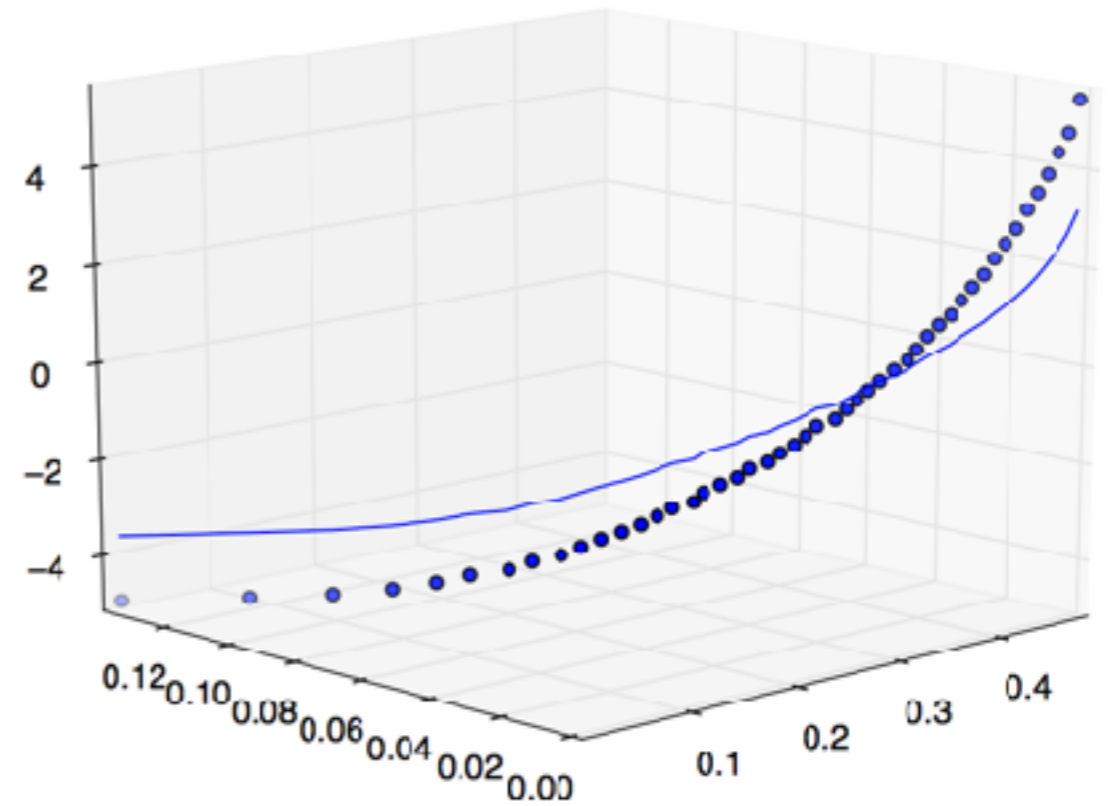
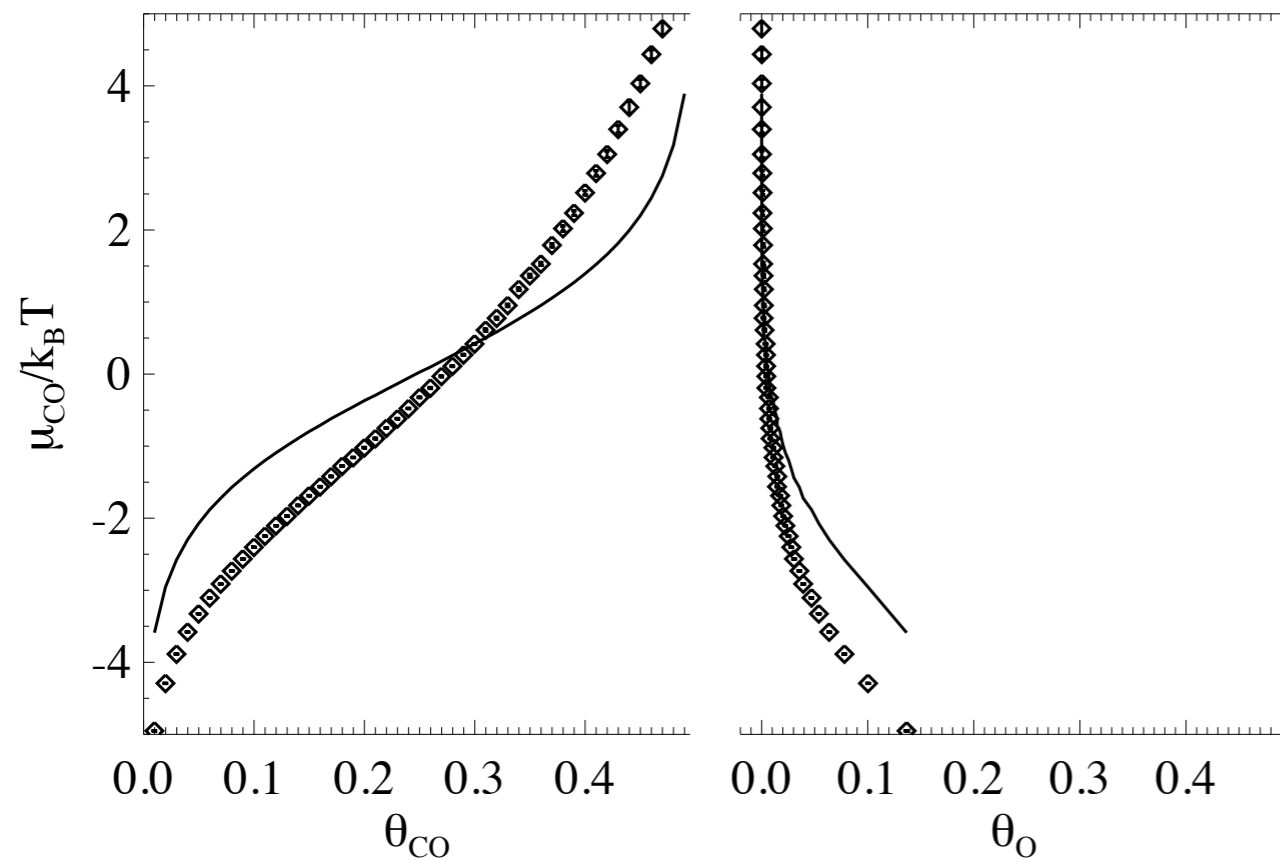
$$\sigma_{\text{CO}} = \lim_{t \rightarrow \infty} (2dt)^{-1} |R_{\text{CO}}(t) - R_{\text{CO}}(0)|^2$$

from KMC, Einstein's relation

$$\sigma_{\text{CO}}/h_{\text{CO}} = 1 - 2\theta_{\text{CO}} - 2\theta_0$$

analytic approximation

# The chemical potential of CO in mixed layers

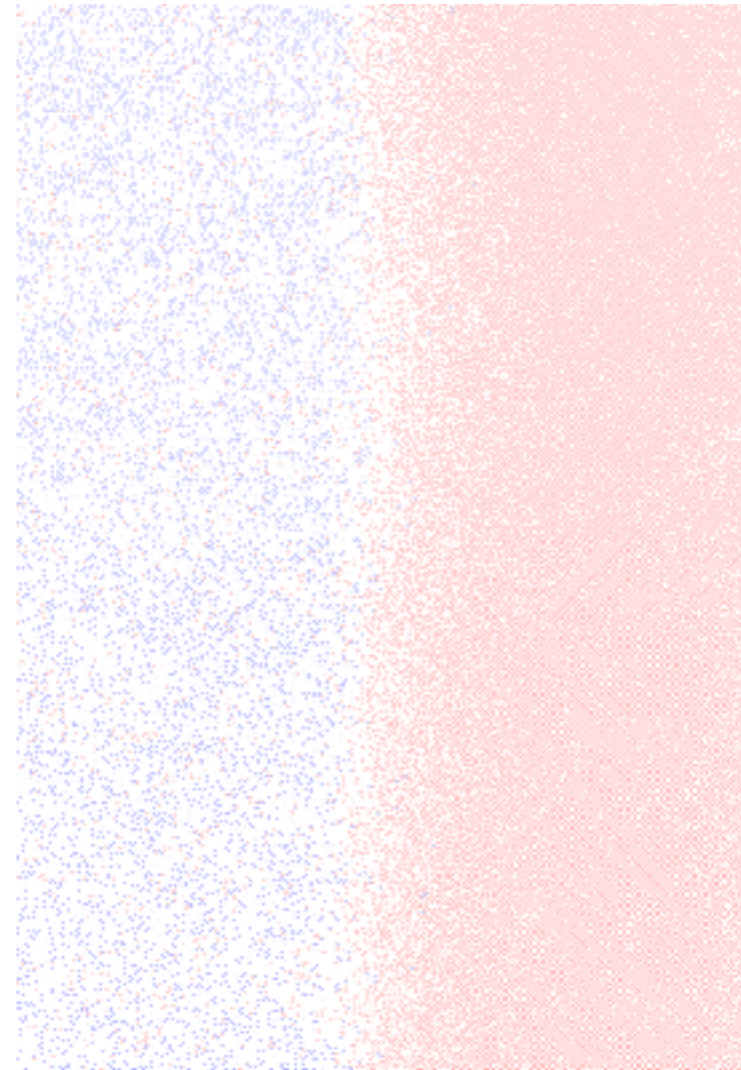
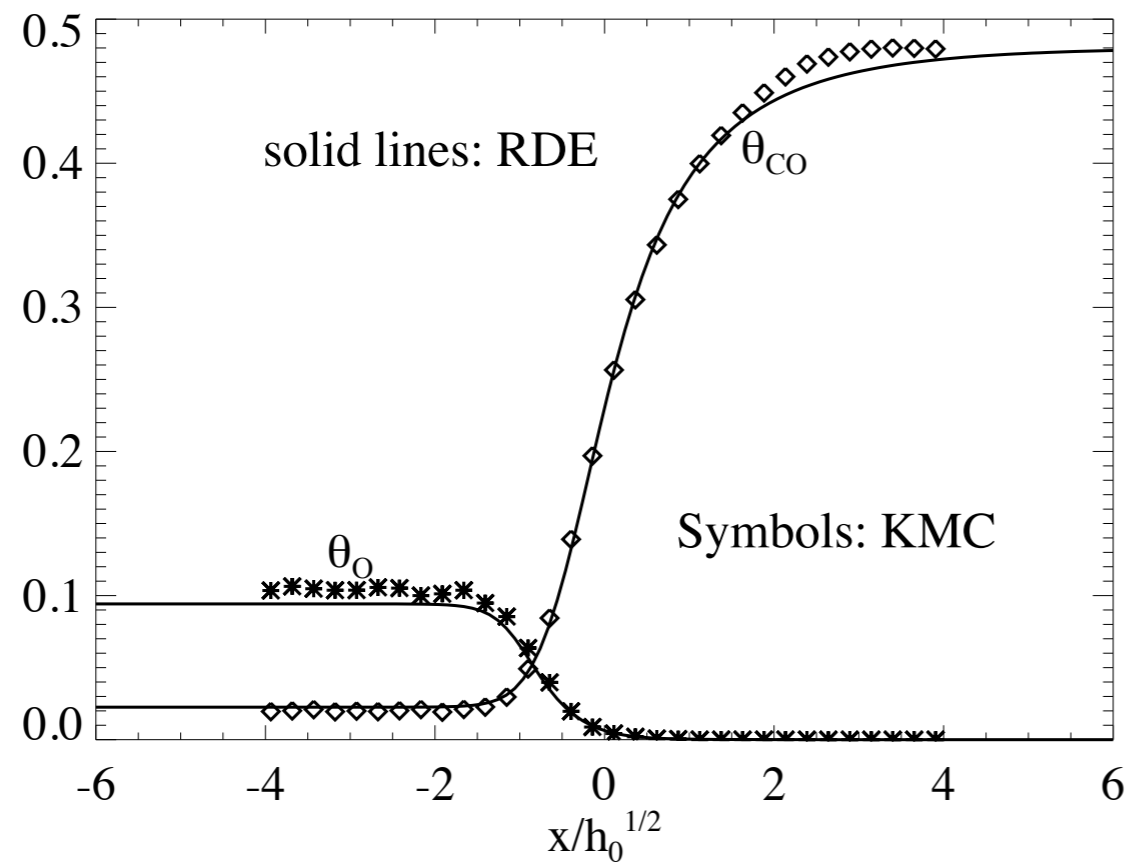


$$\mu_{\text{CO}}/(k_B T) = \ln \theta_{\text{CO}} - \ln [\langle \exp(-\delta E/(k_B T)) \rangle] \quad \text{from KMC, Widom's insertion method}$$

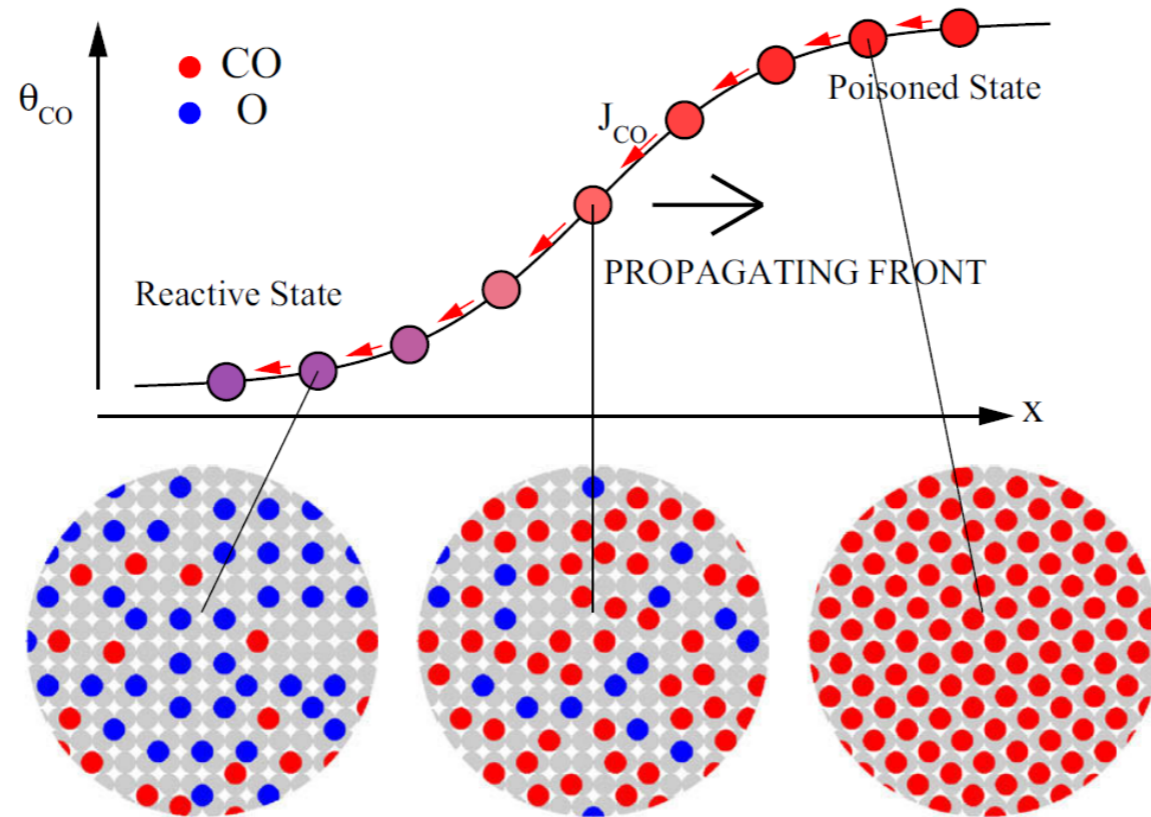
$$\mu_{\text{CO}}/(k_B T) = \ln[2\theta_{\text{CO}}/(1 - 2\theta_{\text{CO}} - 2\theta_{\text{O}})] \quad \text{analytic approximation}$$

# Profile of a chemical Wave

Reaction-Diffusion Eq. vs. Kinetic MC Simulations



# Heterogeneous Coupled Lattice Gas Model



$$\partial \theta_{CO} / \partial t = R_{CO}(\theta_{CO}, \{O\}) - \nabla \cdot \mathbf{J}_{CO}$$

$$\mathbf{J}_{CO} = - (kT)^{-1} \sigma_{CO} \theta_{CO} \nabla \mu_{CO}(\theta_{CO}, \{O\})$$

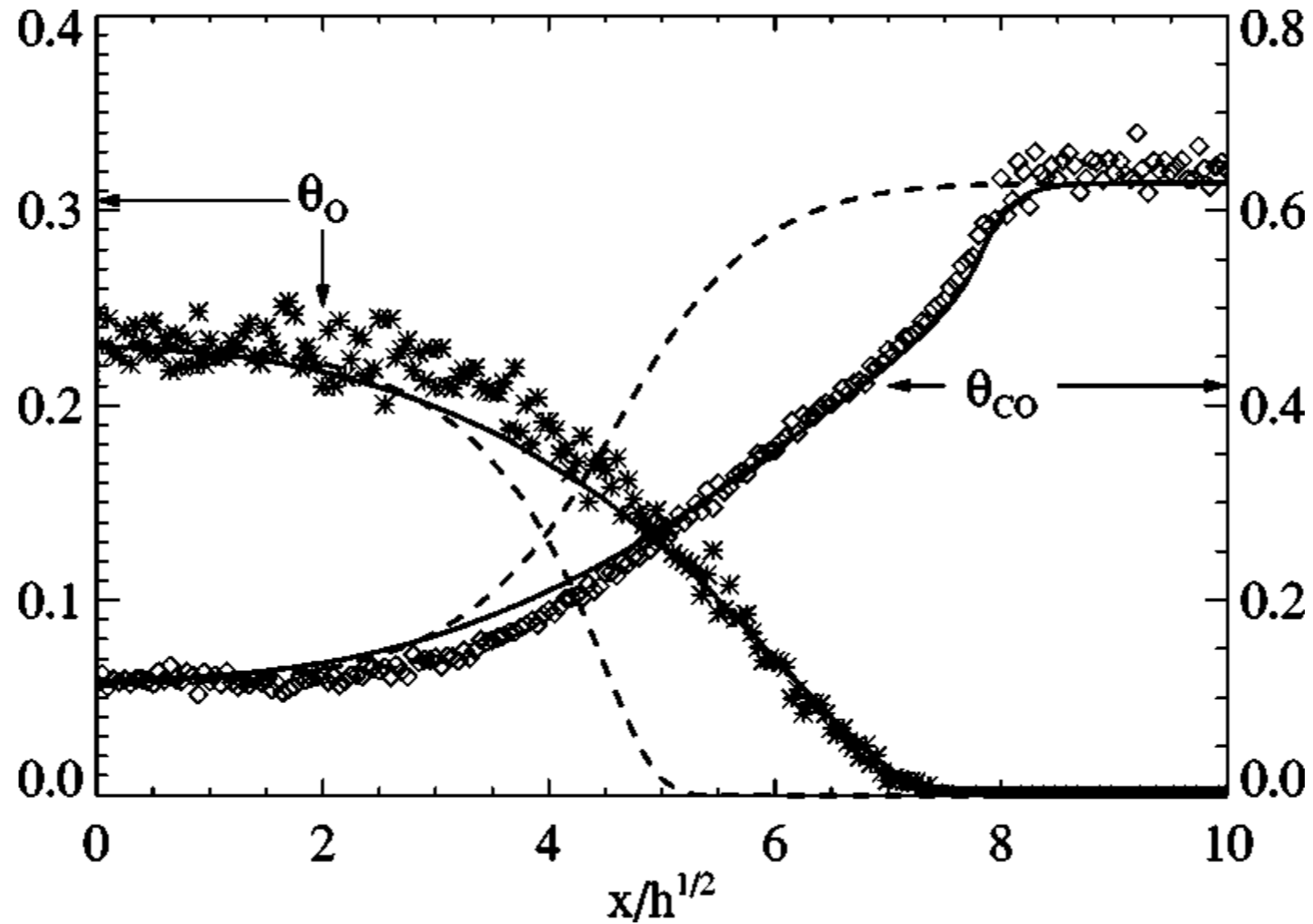
Applications requires:

Availability of massively parallel processors

*Ab initio* calculations of diffusion barriers

# Profile of Stationary Chemical Wave

Realistic Model with Metropolis-like rates for diffusion

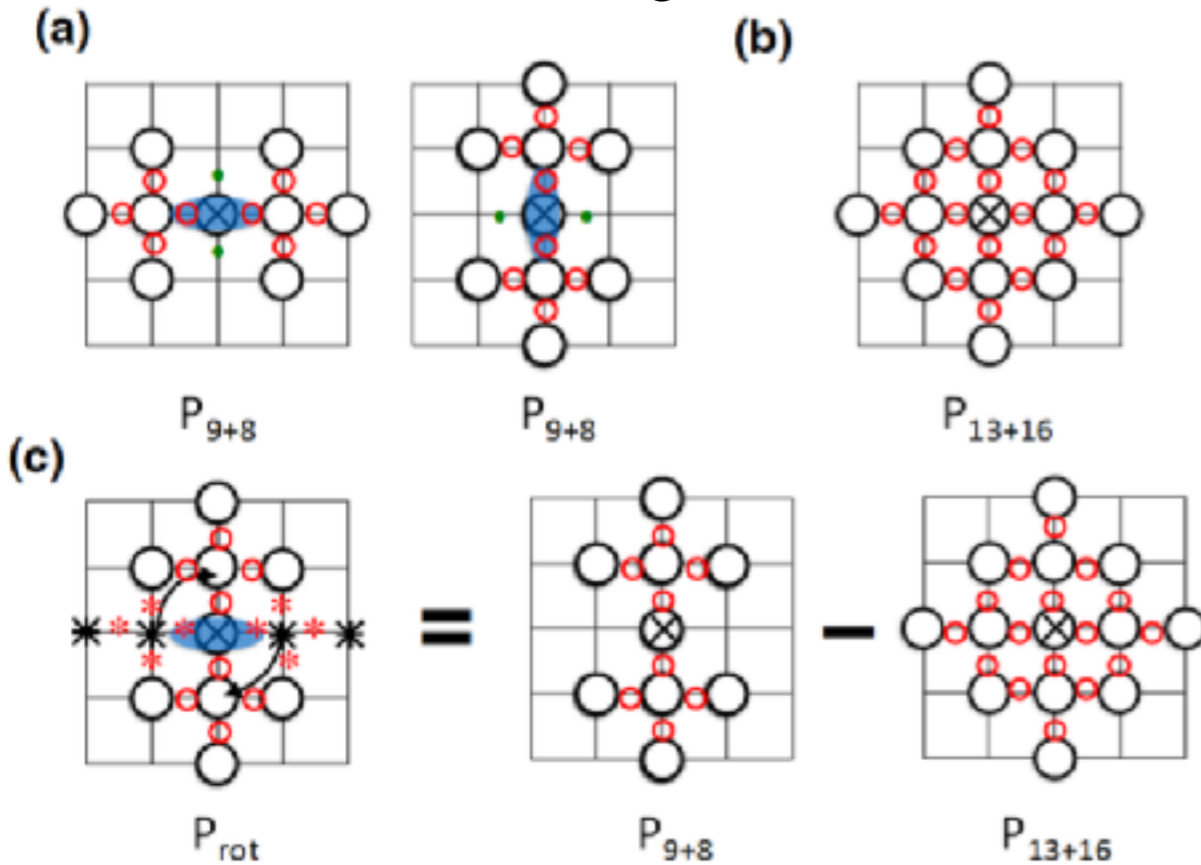


# Summary

- First-principle derived model for surface reaction.
- Multi-site Lattice-gas model, with efficient algorithms and capping technique,  $10^6$  adsorption sites,  $10^3$  seconds.
- Homogenous properties, stationary patterns can be simulated with single processor.
- Spatial-temporal behavior more challenging, HCLG most suitable for massively parallel computer (GPU, Xeon Phi)

## O<sub>2</sub> dissociation

9 4fh + 8 bri blocking, reorientation



$$S_{\text{CO}} = [1 - 2\theta_{\text{CO}} - 2\theta_{\text{O}}][1 - \alpha\theta_{\text{CO}}^4] \\ / [1 - 2\theta_{\text{CO}} - 2\theta_{\text{O}} + 9.5\theta_{\text{CO}}^3 + \theta_{\text{CO}}^2\theta_{\text{O}} + 2\theta_{\text{O}}^6]$$

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$$Q_{8|9} = Q_4^2 Q_2 / Q_1^2$$

$$Q_n = 1 - n\theta_{\text{CO}}^{\text{br}}(\text{loc})$$

$$\theta_{\text{CO}}^{\text{br}}(\text{loc}) = \theta_{\text{CO}}^{\text{br}} / (1 - 2\theta_{\text{O}})$$

