Scattering of Flocks for the Attraction-Repulsion Model

Dieter Armbruster, Stephan Martin, Sebasitan Motsch, Andrea Thatcher

Attraction Repulsion Model

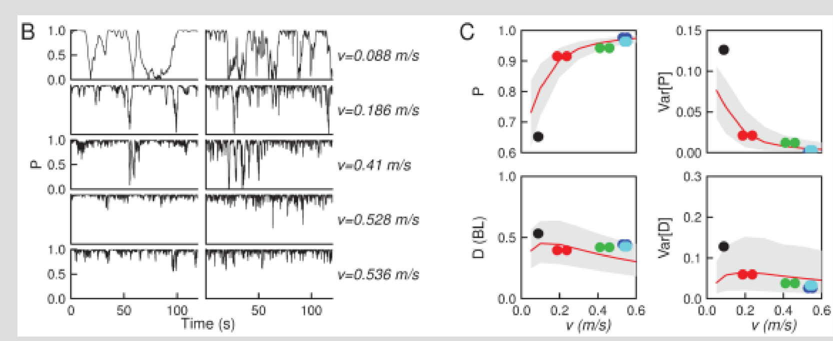
$$\dot{x}_i = v_i$$
$$\dot{v}_i = (\alpha - |v_i|^2)v_i \; - \; \lambda \, \nabla_{x_i} \sum_{j \neq i} U(x_i - x_j)$$
$$U(r) = \; C \exp\left[-\frac{|r|}{l}\right] - \exp[-|r|] \qquad (1)$$

Abstract

We perform scattering experiments for two particle based flocks. We prepare two uniformly translating flocks and direct them at different angles. For two particles, there are two fundamentally different dynamic outcomes: high speed case, the two particles diverge; low speed case, the two particles oscillate and merge. For N particle flocks a similar transition occurs, but trapped solutions are seen at slower speeds

Motivation

Lower speed => Lower orientation order



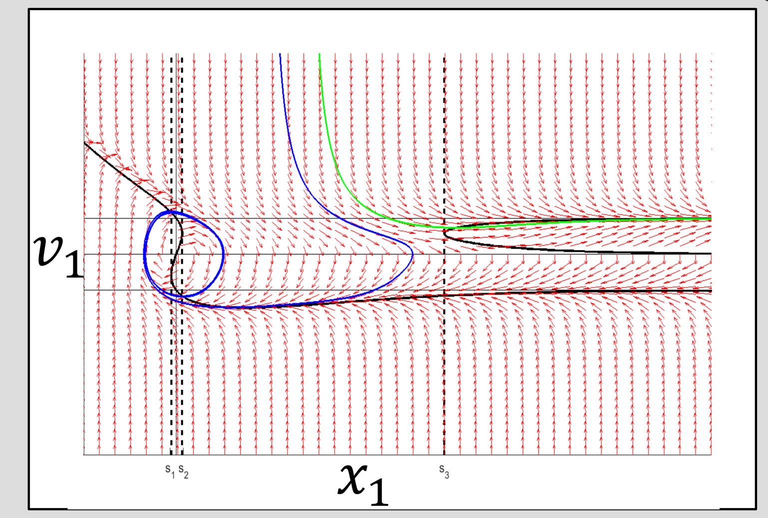
[1]

v fish speed (body lengths/sec)
D Ave Distance between fish
P polarization

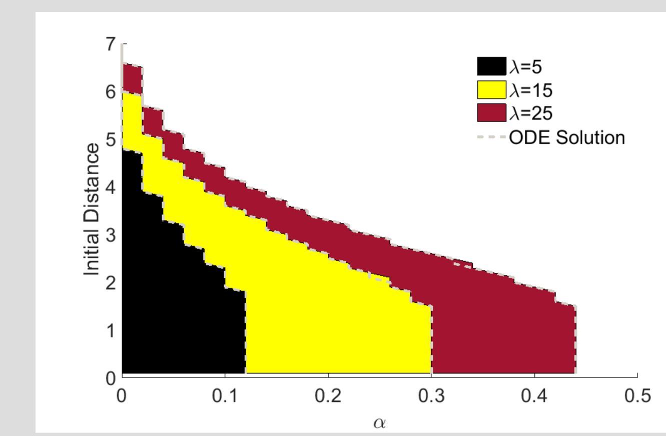
1D

- Supercritical Hopf B
- Diverges

Fixed α and λ

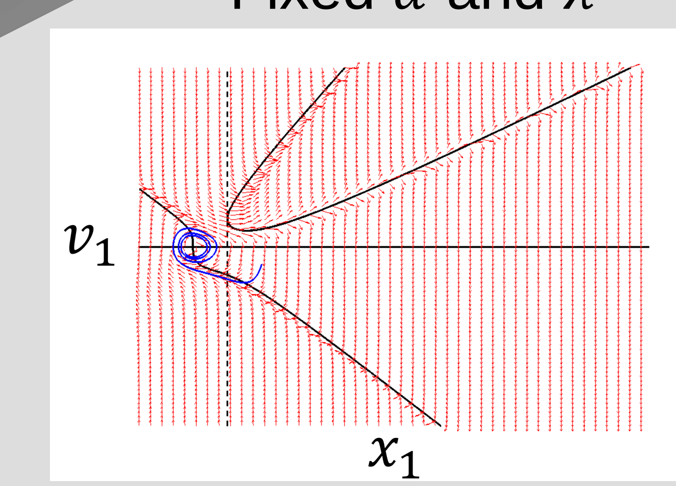


Separatrix



Shaded Area converges to limit cycle

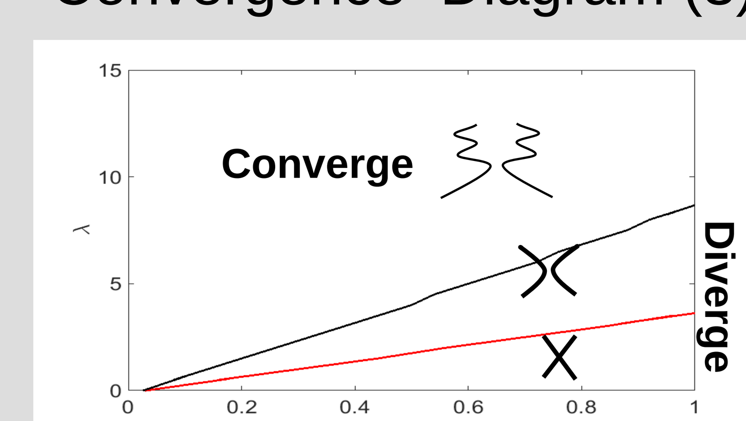
Fixed α and λ



- Center Man. Red. (O(4))
- Local Stability

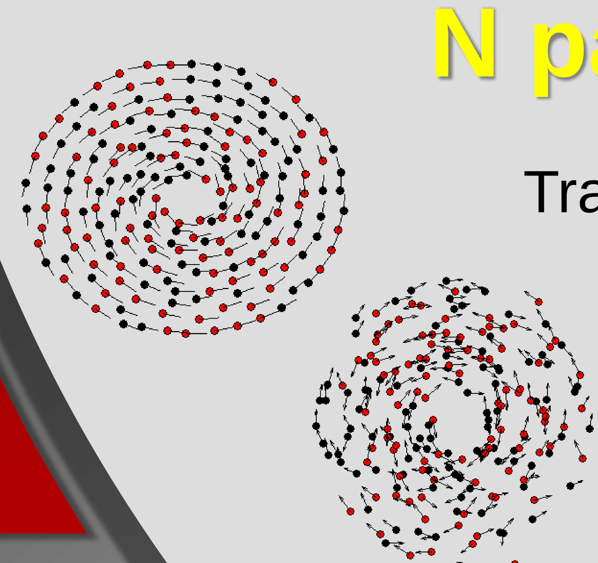
2D

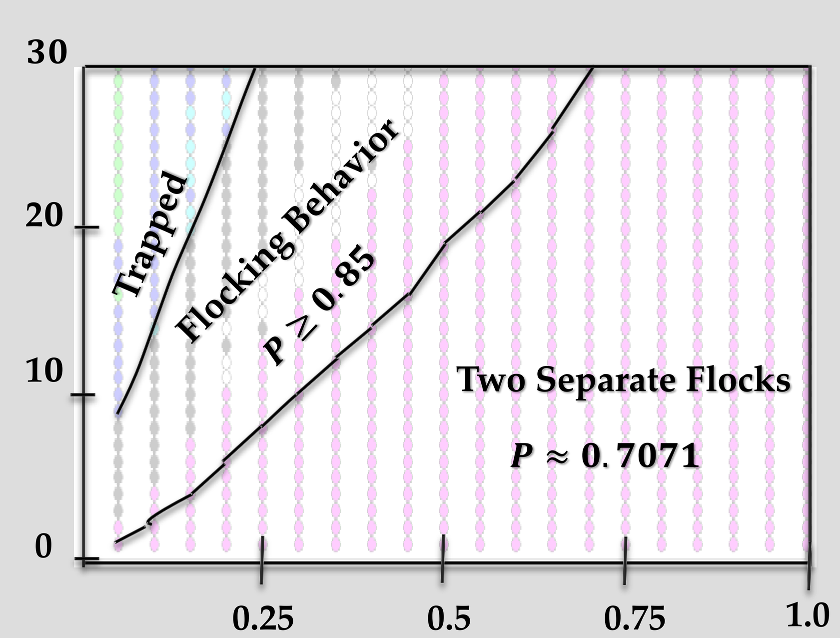
Convergence Diagram (3)



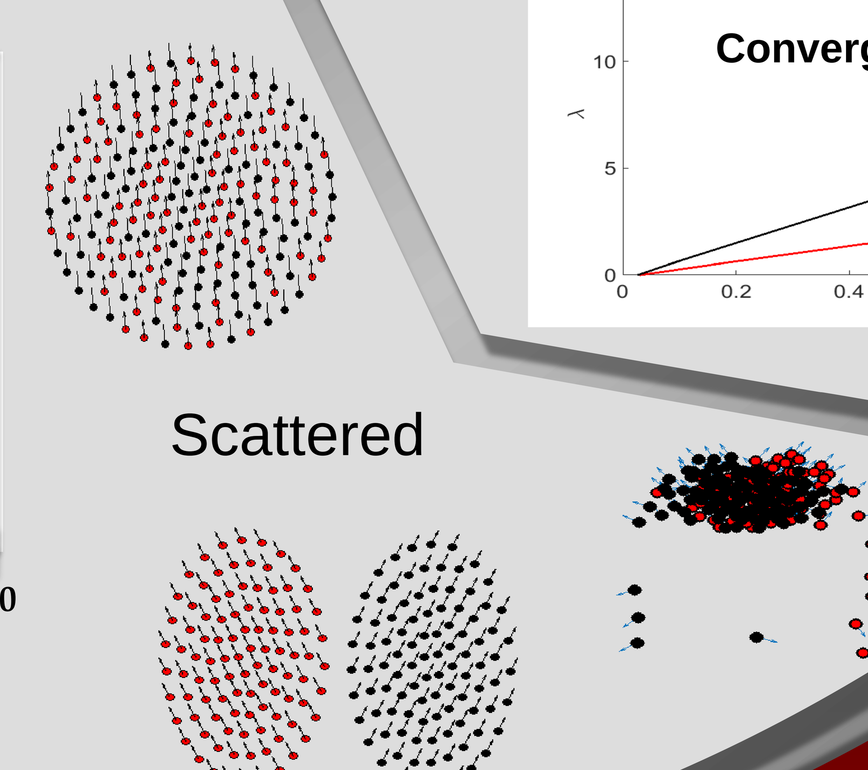
N particles

Trapped

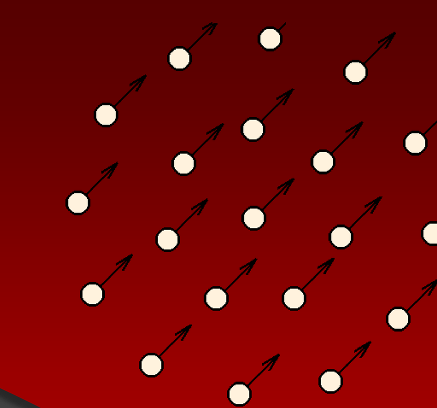




Scattered



Flock



Flocking Solution:

$$(x_i(t), v_i(t)) = (\hat{x}_i + m_0 t, m_0)$$

Where:

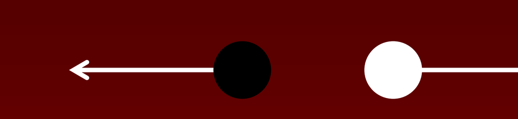
$$\nabla_{\hat{x}_i} \sum_{j \neq 0} U(\hat{x}) = 0 \qquad |m_0| = \sqrt{\alpha}$$

One Particle Flock

Define

$$X = x_1 - x_2$$
$$V = v_1 - v_2$$
$$v = (v_1 + v_2)/2$$

(1D)



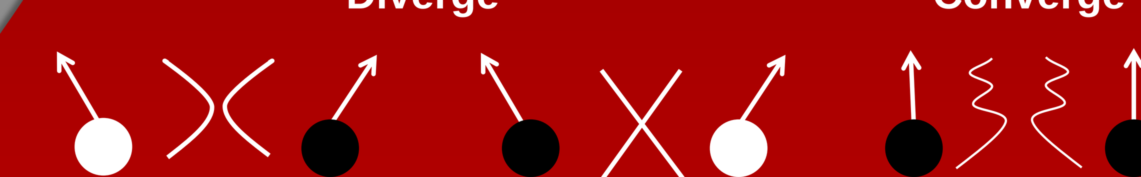
$$\dot{X}_1 = V_1 \qquad (2)$$
$$\dot{V}_1 = \left(\alpha - \left(\frac{V_1^2}{4}\right)\right)V_1 - \frac{2\lambda X_1}{|X_1|}U'$$

Diverge   Converge

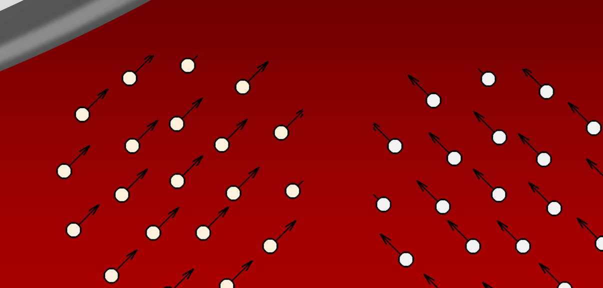
(2D)

$$\dot{X}_1 = V_1$$
$$\dot{V}_1 = \left(\alpha - \left(\frac{V_1^2}{4} + v_2^2\right)\right)V_1 \; - \frac{2X_1}{|X_1|}\lambda U'$$
$$\dot{v}_2 = \left(\alpha - \left(\frac{V_1^2}{4} + v_2^2\right)\right)v_2 \qquad (3)$$

Diverge   Converge



N Particle Flock



Trapped	Scattered
Localized: $P \ll 1$, $M_{abs} \approx 1$	**NonLocalized:** $P \approx 1$, $M_{abs} < 1$

References

Measures

Polarization: $P(t) = \left|\frac{\sum_i v_i(t)}{\sum_i |v_i(t)|}\right|$

Angular Momentum: $M(t) = \frac{|\sum_i r_i(t) \times v_i(t)|}{\sum_i |r_i(t)||v_i(t)|}$

Abs Angular Momentum: $M_{abs}(t) = \left|\frac{|\sum_i r_i(t) \times v_i(t)|}{\sum_i |r_i(t)||v_i(t)|}\right|$

$r_i = x_i - x_{cm}$

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ASU

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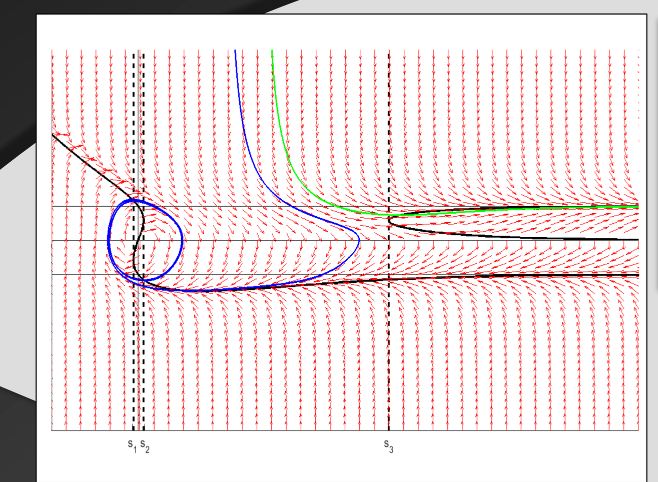
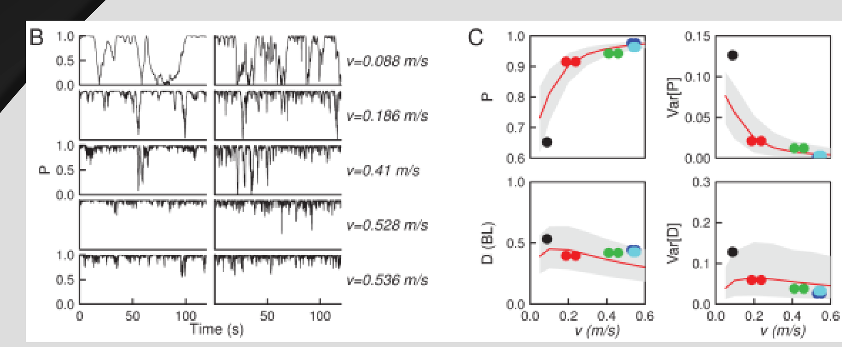
$$U(r) = C \exp\left[-\frac{|r|}{l}\right] - \exp[-|r|]$$



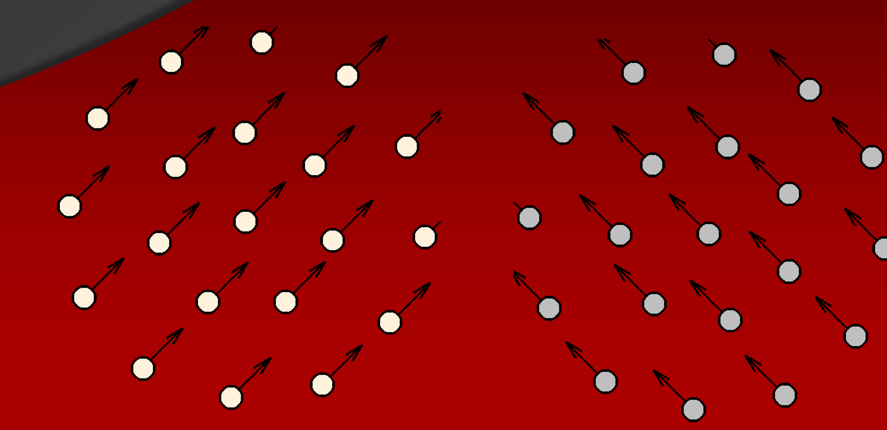
Motivation

The experimental observation that the orientational order is lower when the swimming speed is lower, and is better in faster groups



v fish speed (body lengths/sec)
P polarization
D Ave Distance between fish

Two Flocks



Flocking Solution:

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Where:

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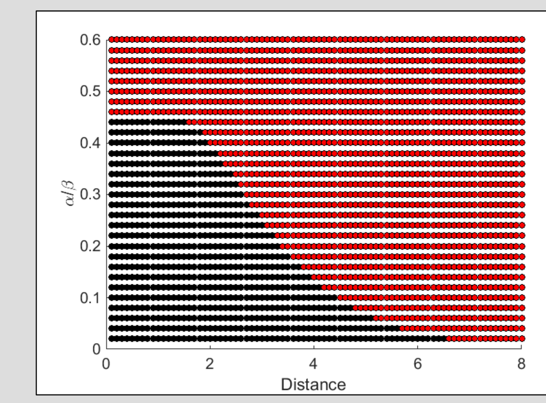
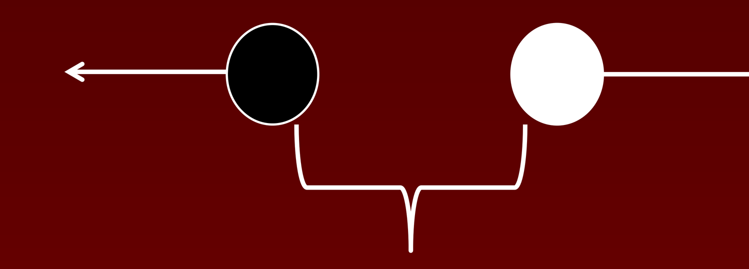


Two Particles (1D)



X $= x_1 - x_2$

V $= v_1 - v_2$

$v = (v_1 + v_2)/2$

$$\dot{X}_1 = V_1$$
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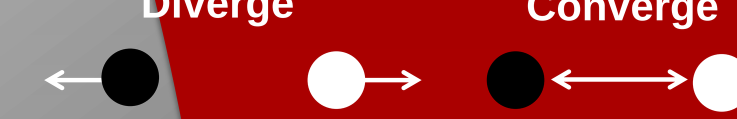
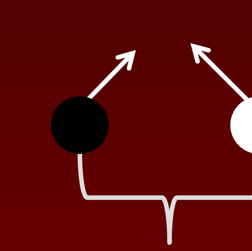


Phase Portrait
Lambda vs alpha

(2D)



$$\dot{X}_1 = V_1$$
$$\dot{V}_1 = \left(\alpha - \beta\left(\frac{V_1^2}{4} + v_2^2\right)\right)V_1 \; - 2\lambda U'$$
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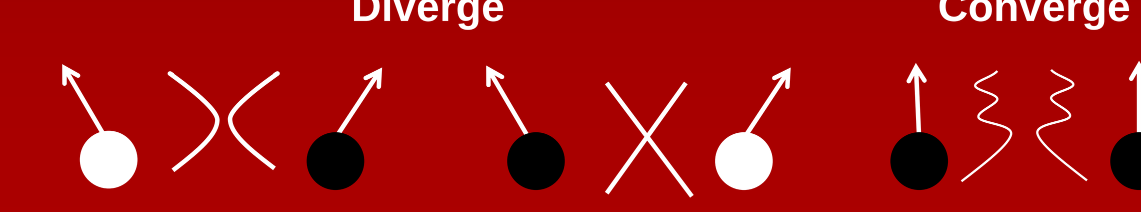
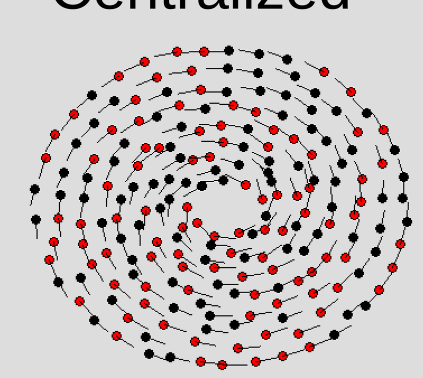


Scattered and Trapped

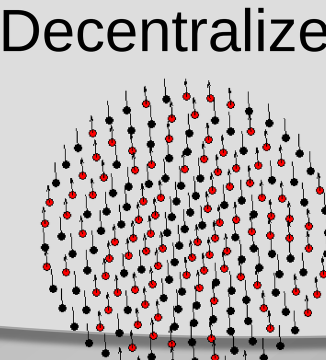
References

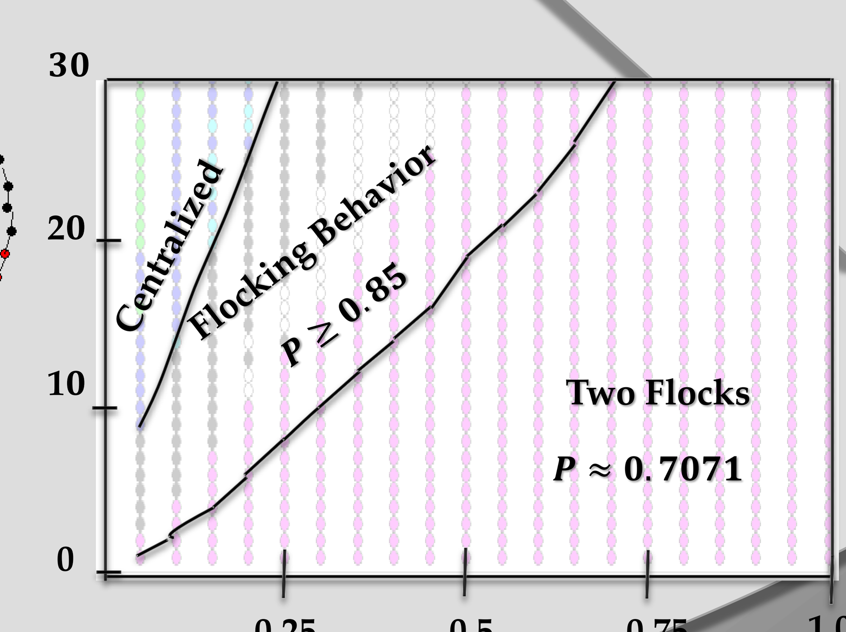
Escape vel vs N

Centralized



Decentralized





N Particles

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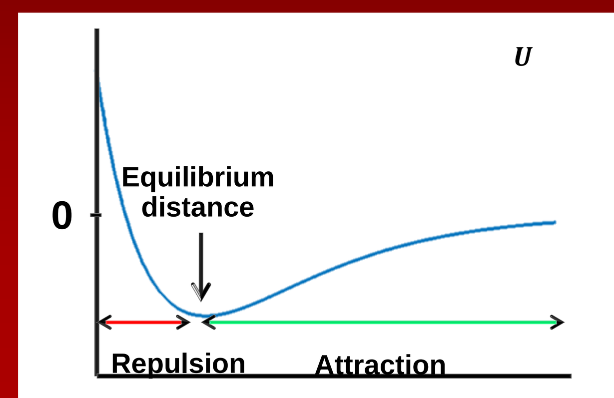

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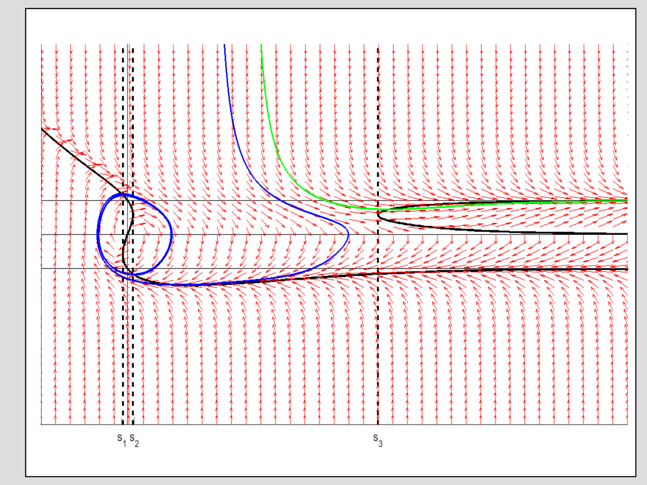

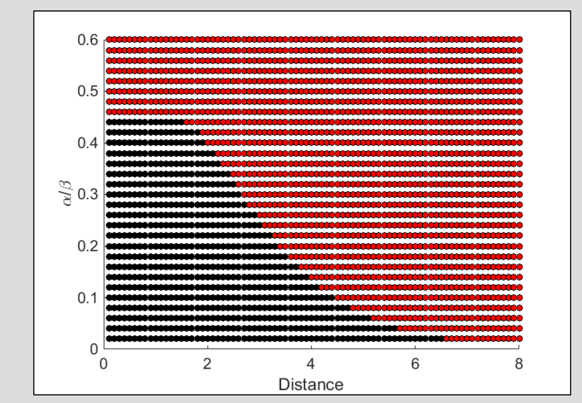

Phase Portrait Lambda vs alpha

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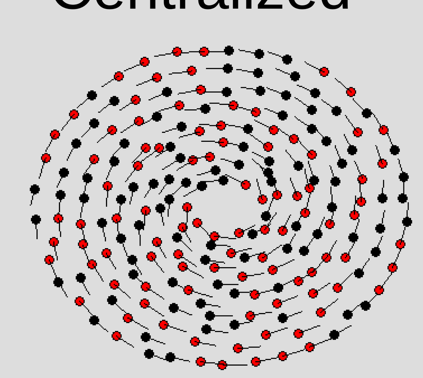

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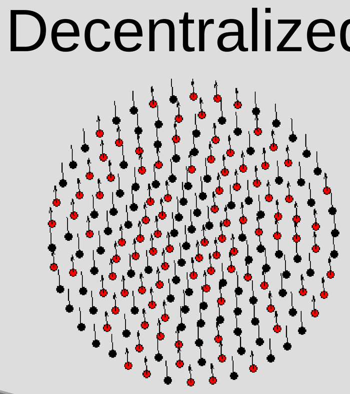
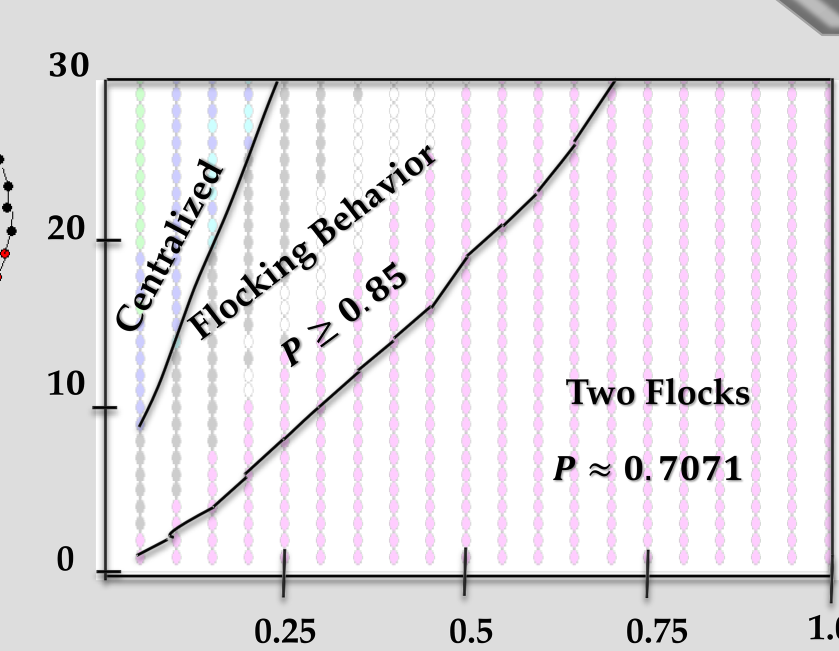
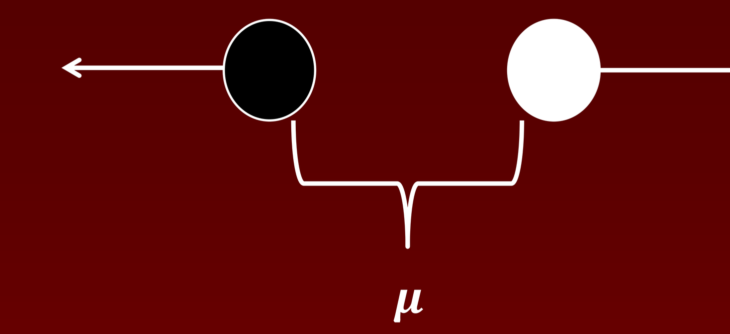




Two Particles (1D)



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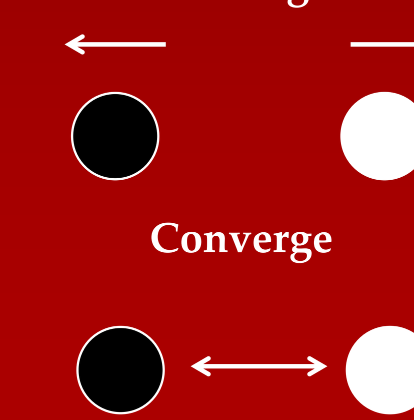
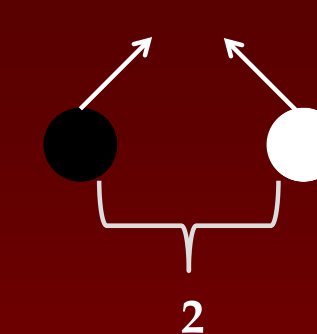


Two Particles (2D)



$$\dot{X}_1 = V_1$$
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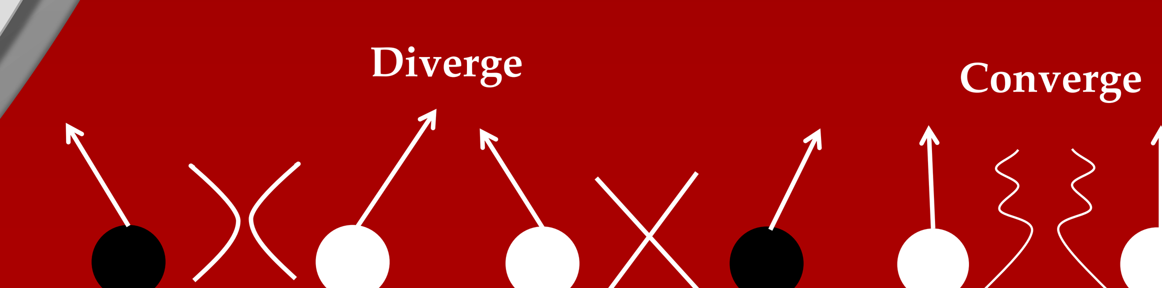
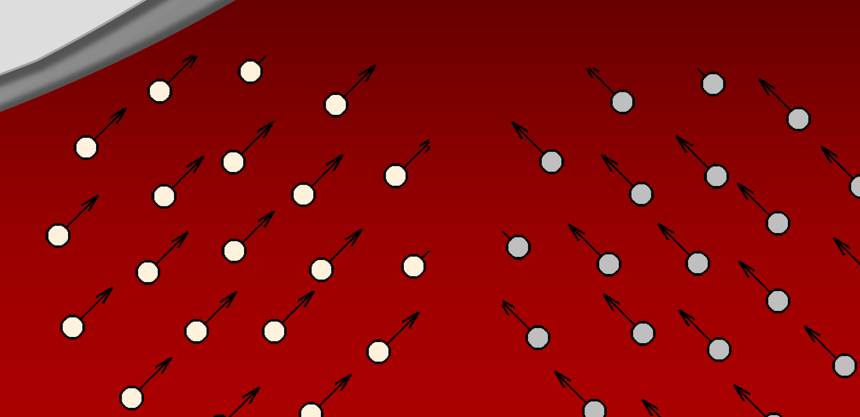
$$\dot{v}_2 = \left(\alpha - \beta\left(\frac{V_1^2}{4} + v_2^2\right)\right) v_2$$



Two Flocks



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