KINETICS DRIVEN BY LOCAL NASH EQUILIBRIA AND RISK AVERSE TRADING STRATEGIES

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INTRODUCTION - CONCEPT

Particles vs. rational agents:

Social or biological agents can behave like

- mechanical particles subject to forces: kinetic theory, minimize a global energy functional.
- rational agents trying to optimize an individual goal, given the behavior of the ensemble: game theory, try to minimize individual cost functions.

Goal:

- Try to reconcile these viewpoints.
- Show that kinetic theory can deal with rational agents.
- Incorporate time-dynamics in game theory.

Applications:

– Social herding behavior: (Degond , Liu, C.R; J Nonlinear Sci. 2014)

– Economics: (Degond , Liu, C.R; J. Stat. Phys. 2014 and Phil. Trans. R. Soc, to appear)

OUTLINE

1. Kinetics vs. game theory.
   - General framework.
   - Differences and similarities; mean field models; non - atomic anonymous games; hydrodynamics.
2. Wealth distribution I:
   - Strategies $\Longleftrightarrow$ Wealth
   - Conservative economies (opinion formation models).
   - Standard hydrodynamics with 'high energy tails' (Pareto tails).
3. Wealth distribution II:
   - Non - conservative systems.
   - Mean field models and strategies.
   - Macroscopic balance laws and generalized collision invariants.
4. Conclusions and outlook.

Kinetics vs. game theory

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Nash equilibria vs. energy minimization $_{02}$

Game with a finite number of players:

- N players $n = 1, \ldots, N$
- Each player can play a strategy $y_n, \; n = 1 : N, \; Y = (y_1, .., y_N)$ in a strategy space $\mathcal{Y}$.
- The cost function (=-payoff) of player n playing strategy y_n in the presence of the other players playing strategy $\hat{Y}_n = (y_1, \ldots, y_{n-1}, y_{n+1}, \ldots, y_N)$ is $\phi_n(y_n, \hat{Y}_n)$
- Each player tries to minimize its cost function by acting on their strategy y_n, not touching the others' strategies $\hat{Y}_n$ (Best response strategy).

Nash equilibrium:

Strategy $Y = (Y_1, \ldots, Y_N)$ such that no player can improve on its cost function by acting on its own strategy variable y_n.

$$\boxed{y_n \to \psi_n(\hat{Y}_n), \; \phi_n(\psi_n, \hat{Y}_n) = \min_z \phi_n(z, \hat{Y}_n), \; n = 1 : N}$$

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Nash equilibrium $\Longleftrightarrow$ Fixed point problem

$$\boxed{y_n = \psi_n(\hat{Y}),\ n = 1:N\,.}$$

This is different from minimizing a global energy functional or $\sum_n \phi_n$ (prisoner's dilemma).

Identical players and anonymous games:
$\phi_n(y_n, \hat{Y}_n) = \phi(y_n, \hat{Y}_n)$.

- Players with the same strategy cannot be distinguished.

THE CONTINUUM MODEL 06

$$\phi_n(y_n, \hat{Y}_n) = \phi(y_n, \hat{Y}_n) \Rightarrow \phi(y_n, \hat{Y}_n) \to \phi_f(y)$$

Mean field model using a mean field cost function $\phi_f(y)$, dependent on the distribution of strategies $f(y)\, dy$.

Nash equilibrium:

$$\int \phi_{f_{NE}}(y) f_{NE}\, dy = \inf_f \int \phi_{f_{NE}}(y) f\, dy$$

General framework of Non-Cooperative, Non-Atomic, Anonymous games with a Continuum of Players (NCNAACP)

References:

- Aumann, Mas Colell, Schmeidler, Shapiro & Shapley,
- Mean-field games: Lasry & Lions, Cardaliaguet

CONTINUUM MODEL WITH MIXED STRATEGIES

The basic model:

Each (identical) player tries to march towards its Nash equilibrium (i.e. in the direction of $-\nabla_y \phi(y; f)$) at each time step. $\Rightarrow$ **kinetic equation with state dependent cost function $\phi_f(y)$ (best reply strategy, open loop control).**

$$\boxed{\partial_t f(y,t) - \nabla_y \cdot [f \nabla_y \phi_f(y)] = 0}$$

Noise: 08

In a game with mixed strategies the choice of y is not certain but the player picks y with some randomness. This is generally modeled by a Brownian motion term of the form

$$\boxed{dy_n = -\nabla_{y_n} \phi(y_n, \hat{Y}_n)\, dt \; + \sqrt{2D} B_t}$$

which results in the kinetic equation

$$\boxed{\partial_t f(y,t) - \nabla_y \cdot [f \nabla_y \phi f(y)] = D \Delta_y f}$$

STEADY STATE FOR NE DRIVEN KINETICS

Show that the equilibrium of the kinetic equation is indeed a NE.

The equilibrium is given by

$$Q(f) = \nabla_y \cdot [f \nabla_y \phi(y; f) + D \nabla_y f] = 0$$

- The solution of $Q(f) = 0$ can be reformulated as a fixed point problem.
- For a given ϕ, $Q(f)$ is linear in f.
- So, we write $Q(f) = \mathfrak{C}(f, \phi)$ with $\mathfrak{C}$ bilinear in f and ϕ.

For a given ϕ, the solution of $C(f, \phi) = 0$ is given by

$$f(y) = \frac{\rho}{Z_\phi} e^{-\phi/D}, \quad Z_\phi = \int e^{-\phi(y)/D} \, dy$$

for an arbitrary parameter (the number of agents) $\rho = \int f(y) \, dy$.

- The solution of $Q(f) = \mathfrak{C}(f, \phi_f) = 0$ is given by $\boxed{f(y) = \rho g(y), \ \forall \rho}$ with g the Gibbs measure

FIXED POINT PROBLEM FOR THE GIBBS MEASURE

The fixed point problem is of the form

$$\boxed{g(y) = \frac{1}{Z_{\phi_g}} e^{-\phi_g/D},\ Z_{\phi_g} = \int e^{-\phi_g(y)/D}\, dy\ ,}$$

with the normalized Gibbs measure g satisfying $\boxed{\int g(y)\, dy = 1,\ \forall x}$.

NASH EQUILIBRIUM 11

Nash equilibrium:

$$\int \mu_{f_{NE}}(y) f_{NE}(y)\, dy = \inf_f \int \mu_{f_{NE}}(y) f(y)\, dy$$

Theorem (Degond, Liu,CR, 2013)

The Gibbs measure g given by the fixed point problem

$$g(y) = \frac{1}{Z_{\phi_g}} e^{-\phi_g/D},\; Z_{\phi_g} = \int e^{-\phi_g(y)/D}\, dy\;,$$

is a Nash equilibrium for the modified cost function

$$\mu_f(y) = \phi_f(y) + D \ln f(y)$$

CONSEQUENCE

The equation

$$\boxed{\partial_t f = \nabla_y \cdot [f \nabla_y \phi_f + D \nabla_y f]}$$

models the interaction of an ensemble of agents (under an IID assumption), each marching towards a Nash equilibrium in infinitesimal time steps.

- Different from mean field game theory, where players optimize strategy over a finite time horizon.

MOMENT DEPENDENT COST FUNCTIONS

The special case when ϕ_f depends on f only through the first K normalized moments

$$\phi_f = \phi_{\rho_f, \vec{\Upsilon}_f}$$

$$\boxed{\rho_f = \int f\, dy,\ \ \vec{\Upsilon}_f = (\Upsilon_1(f), .., \Upsilon_K(f)),\ \ \Upsilon_k(f) = \frac{\int y^k f\, dy}{\int f\, dy}}$$

- Yields a nonlinear operator $Q(f) = \mathfrak{C}(f, \phi_{\rho_f, \vec{\Upsilon}_f})$, whose nonlinearity is given only via the moments $\rho_f, \vec{\Upsilon}_f$.
- **In this case, the infinite dimensional fixed point problem,defining the Gibbs measure, reduces to a finite dimensional fixed point problem for the vector $\vec{\Upsilon}$.**

- In this case, the infinite dimensional fixed point problem,defining the Gibbs measure, reduces to a **finite dimensional** fixed point problem for the vector $\vec{\Upsilon}$.

$$g(y) = \frac{1}{Z_{\phi_{1,\vec{\Upsilon}}}} e^{-\phi_{1,\vec{\Upsilon}}/D}, \; Z_{\phi_{1,\vec{\Upsilon}}} = \int e^{-\phi_{1,\vec{\Upsilon}}(y)/D}\, dy \, ,$$

$$\boxed{\vec{\Upsilon} = \frac{1}{Z_{\phi_{1,\vec{\Upsilon}}}} \int \begin{pmatrix} y \\ . \\ . \\ y^K \end{pmatrix} e^{-\phi_{1,\vec{\Upsilon}}/D}\, dy, \; Z_{\phi_{1,\vec{\Upsilon}}} = \int e^{-\phi_{1,\vec{\Upsilon}}(y)/D}\, dy \, ,}$$

Games with configuration variables 15

- Add configuration (aka "type") variable $X = (x_1, .., x_N)$ (e.g. space)
- x can be real space, the propensity to trade etc.
- Motion depends on both type X and strategy Y

$$\boxed{\dot{x}_n = V(x_n, y_n), \qquad n = 1:N}$$

- Cost function depends also on types X

$$dy_n(t) = -\nabla_{y_n}\phi(y_n, \hat{Y}_N, X)\,dt + \sqrt{2d}\,dB_t, \qquad n = 1:N$$

Probability distribution depends on type x and strategy y:
$\boxed{f = f(x, y, t)}$.
Satisfies space-dependent kinetic equation.:

$$\boxed{\partial_t f + \nabla_x \cdot (V(x,y)f) - \nabla_y \cdot \left(\nabla_y \phi_f f\right) - D\Delta_y f = 0}$$

with $\phi_f = \phi_{f(t)}(x, y)$

SCALE SEPARATION AND HYDRODYNAMIC CLOSURES

- Kinetic theory provides large time macroscopic limits for different time scales.
- Assume that the evolution of the strategy y is much faster than that of type x.
- Fast equilibration of strategy leads to slow evolution of type

$$\boxed{\partial_t f + \nabla_x \cdot [V(x,y)f] = \frac{1}{\varepsilon} Q(f) = \frac{1}{\varepsilon} \nabla_y [f \nabla_y \phi_f(x,y,t) + D \nabla_y f]}$$

ε: ratio of evolution time scales.

In zero'th order the solution will live on the manifold given by $Q(f) = 0$, parameterized by a finite set of $x-$ dependent parameters.

THE GIBBS MEASURE AND THE SOLUTION OF $Q(F) = 0$

Standard Approach:

- Assume the solution of the fixed point problem

$$\boxed{g(y) = \frac{1}{Z_{\phi_g}} e^{-\phi_g/D}, \; Z_{\phi_g} = \int e^{-\phi_g(y)/D} \, dy \,,}$$

  depends on K local parameters $S = (s_1, .., s_K)$. Therefore $g = g(x, y; S)$
- Assume that, in addition to $y = 1$, there are K collision invariants $C = (c_1, .., c_K)$ of Q such that

$$\int \begin{pmatrix} 1 \\ c_1(y) \\ .. \\ c_K(y) \end{pmatrix} Q(f) \, dy = 0, \;\; \forall f$$

  holds.
- Parameterize the solution of $Q(f) = 0$ by its moments C and close the moment equations.

This gives $K+1$ conservation laws of the form

$$\partial_t[\rho \int \begin{pmatrix} 1 \\ C(y) \end{pmatrix} g(x,y;S)\, dy] + \nabla_x \cdot [\int V(x,y)\rho \begin{pmatrix} 1 \\ C(y) \end{pmatrix} g(x,y;S)\, dy] = 0$$

This gives $K+1$ macroscopic conservation laws for the $K+1$ macro-variables $\rho(x), S(x)$.

- **Problem: What happens if there are fewer than K collision invariants?**
- The local equilibrium $f_{loc}(x,y,t) = \rho(x,t)g(x,y,S(x,t))$ depends on K parameters S, but there are only L conserved quantities $C = (c_1(y),..,c_L(y))$ with $L < K$?
- Leads to the concept of Generalized Collision Invariants (GCI), (Degond & Motsch; 2009).

RELATION TO MEAN FIELD GAMES (Lasry & Lions) 22

- Mean-field game approach directly provides continuum equations without Kinetic Eq. step.
- Relies on an optimal control approach within a finite horizon time $[0, T]$ using the Hamilton - Jacobi - Bellman system.

$$\partial_t \rho + \nabla_x \cdot (V\rho - mDu) = D\Delta\rho, \ , \rho(x,0) = \rho_I(x)$$
$$\partial_t u = \frac{1}{2}|\nabla u|^2 - D\Delta u + \nabla_x \phi(x,\rho), \ u(x,T) = 0$$

- u is a control corresponding to agents' mean strategy at x. (Plays the role of the parameter S in the kinetic theory.)
- Optimizes not only the local cost in time, but the cost along a particle path $x(t), \ t \in [0, T]$.
- Infinite dimensional two point boundary value problem for $t \in [0, T]$.

In special cases the hydrodynamic system, arising from the kinetic model is equivalent to the limit $T \to 0$ in the Lasry - Lions model.

SUMMARY PART I:

- Kinetic equation can be interpreted as incremental march towards Nash equilibrium.
- Kinetic equilibria are Nash equilibria if the correct mean field cost function is used.
- Relation to mean field games via infinitesimal time horizon (open loop vs. closed loop control)

OUTLINE:Wealth distribution I

1. Kinetics vs. game theory.
2. Wealth distribution I:
   - Strategies $\Longleftrightarrow$ Wealth
   - Conservative economies (opinion formation models).
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A MODEL OF CONSERVATIVE ECONOMIES 24

Bouchaud & Mézard ; Cordier, Pareschi & Toscani ; Düring & Toscani

$$\boxed{\partial_t f(x,y,t) + \nabla_x \cdot [fV(x,y)] = \frac{1}{\varepsilon}\mathfrak{C}(f, \phi_{\rho_f,\Upsilon_f})}$$

$$\mathfrak{C}[f,\phi] = \partial_y[f\partial_y\phi + \omega\partial_y(y^2 f)]$$

The cost function ϕ depends on f only through its moments!

$$\boxed{\rho_f(x) = \int f(x,y)\,dy,\ \Upsilon_f(x) = \frac{\int f(x,y)\,dy}{\rho_f(x)}}$$

- Note: $y > 0$. Diffusion operator $\partial_y^2(y^2 f)$ associated to geometric Brownian motion.
- In the work of Cordier, Pareschi & Toscani ; Düring & Toscani etc., y is the individual wealth of an agent (identified with a strategy in a game theoretical framework).

The potential $\phi_{\rho,\Upsilon}$ is taken to be a quadratic. ϕ is of the form

$$\boxed{\phi_{\rho,\Upsilon}(x,y) = \frac{\kappa}{2}\frac{\int (y-y')^2 f(x,y')\, dy'}{\int f(x,y')\, dy'} = \frac{\kappa}{2}(y-\Upsilon_f)^2 + const(x)}$$

- Υ_f denotes the local mean wealth.
- Quadratic pairwise interaction potential ϕ_Υ; **models binary trading with the strategy to equalize the wealth.**
- ϕ depends on f only through its moments.

Solving the fixed point problem gives

$$\partial_y[\kappa(y-\Upsilon_g)]g + \omega\partial_y(y^2 g)] = 0, \int g\, dy = 1 \Rightarrow g = const\cdot e^{-\frac{\kappa}{2\omega}(y-\Upsilon)^2}$$

$$\Upsilon = \frac{1}{Z_{\phi_\Upsilon}}\int y e^{-\frac{\kappa}{2\omega}(y-\Upsilon)^2}\, dy,\ Z_{\phi_{\vec{\Upsilon}}}(x) = \int e^{-\frac{\kappa}{2\omega}(y-\Upsilon)^2}\, dy\,,$$

which is a trivial identity.

So, the solution of of $C(f,\phi_f) = 0$ depends on the two macroscopic parameters ρ, Υ (the density of agents and their mean wealth $\Rightarrow K = 1$).

CONSERVATION LAWS

Using geometric Brownian motion, the trading operator C also conserves the mean wealth, i.e.

$$\int \begin{pmatrix} 1 \\ y \end{pmatrix} \partial_y [f\kappa(y - \Upsilon) + \omega \partial_y (y^2 f)] \, dy = 0, \ \forall \rho, \Upsilon$$

This gives a standard hydrodynamic limit for the macroscopic variables of the form

$$\boxed{\partial_t \begin{pmatrix} \rho \\ \rho\Upsilon \end{pmatrix} + \int \begin{pmatrix} 1 \\ y \end{pmatrix} \nabla_x \cdot [V f_{loc}(x, y)] \, dy = 0}$$

with the local equilibrium density f_{loc} given by an inverse $\Gamma-$ distribution:

$$\boxed{f_{loc} = \rho g_\Upsilon, \ g_\Upsilon = \frac{1}{Z_\Upsilon} y^{-\frac{\kappa}{2}-2} e^{-\frac{\kappa\Upsilon}{\omega y}}}$$

- **This can be interpreted as a march of agents towards local Nash equilibria for game associated to cost**

$$\boxed{\mu_{\rho,\Upsilon}(y) = (\kappa + 2\omega)\ln y + \kappa\frac{\bar{\Upsilon}}{y} + \omega \ln \rho}$$

(Degond, Liu,Cr, 2012)

- The local equilibrium $\boxed{g_\Upsilon = \frac{1}{Z_\Upsilon} y^{-\frac{\kappa}{2}-2} e^{-\frac{\kappa\Upsilon}{\omega y}}}$ is an inverse $\Gamma-$ distribution, and has "fat Pareto tails" as $y \to \infty$ (Düring & Toscani, 2007).
- $f(x,y)$ **decays only rationally as** $y \to \infty$**.**

OUTLINE:Wealth distribution II

1. Kinetics vs. game theory.
2. Wealth distribution I:
3. Wealth distribution II:
   - Non - conservative systems.
   - Mean field models and strategies.
   - Macroscopic balance laws and generalized collision invariants.
4. Conclusions and outlook.

NON CONSERVATIVE ECONOMIES 30

Basic concept:

- Agents do not trade with each other individually, but rather with a local market, optimizing the individual wealth w.r.t. the moments of the local wealth in the market.
- Their trading frequency as well as their goals depend on the local value and and risk (uncertainty) of the market (i.e. higher order moments of f).
- $\Rightarrow$ **The total wealth during trading is not conserved.**

The cost function for the individual agent is then given by

$$\phi_n = \phi(x_n, y_n, \vec{\Upsilon}), \ \vec{\Upsilon} = (\Upsilon_1, .., \Upsilon_K), \ \Upsilon_k = \frac{1}{\rho(x_n)} \sum_n y_n^k$$

and in the continuum model

$$\phi_f(x,y) = \phi_{\vec{\Upsilon}_f(x)}(y), \ \vec{\Upsilon}_f(x) = \frac{1}{\rho_f(x)} \int \begin{pmatrix} y \\ . \\ . \\ y^K \end{pmatrix} f(x,y)\, dy$$

HARMONIC POTENTIALS

- As in the previous case, we use a harmonic potential $\phi_{\vec{\Upsilon}}$ of the form

$$\boxed{\phi_{\vec{\Upsilon}}(y) = \frac{a_{\vec{\Upsilon}}}{2}(y - b_{\vec{\Upsilon}})^2}$$

- $a_{\vec{\Upsilon}}$ is the agent's propensity to trade and $b_{\vec{\Upsilon}}$ is its goal.
- **$a_{\vec{\Upsilon}}$ and $b_{\vec{\Upsilon}}$ depend now on higher order moments of f!**
- We consider the case $K = 2,\ \vec{\Upsilon} = \begin{pmatrix} \Upsilon_1 \\ \Upsilon_2 \end{pmatrix}$. (dependence on value and risk of the local market).

RISK AVERSE STRATEGIES 32

- The agent uses the mean Υ_1 and the variance $\Upsilon_2 - \Upsilon_1^2$ of the market worth, i.e. the risk, as its basis for making decisions. So $K = 2, \vec{\Upsilon} = (\Upsilon_1, \Upsilon_2)$.
- We set $\boxed{a_{\vec{\Upsilon}} = \frac{\Upsilon_2}{\Upsilon_2 - \Upsilon_1^2}}$.
- $\frac{1}{a_{\vec{\Upsilon}}} = \frac{\Upsilon_2 - \Upsilon_1^2}{\Upsilon_2}$ is the variation coefficient (dimensionless measure of the uncertainty in the market). **Agent behavior is risk averse!**

Goals:

- Freedom in choosing $b_{\vec{\Upsilon}}$.
- One choice: $\boxed{b_{\vec{\Upsilon}} = (1 + \lambda)\Upsilon_1}$, i.e. the agent tries to beat the market by a factor $1 + \lambda$.

GIBBS MEASURE AND FIXED POINT PROBLEM

The resulting fixed point problem for the Gibbs measure is

$$\boxed{\vec{\Upsilon} = \int \begin{pmatrix} y \\ y^2 \end{pmatrix} g(y)\, dy, \quad g(y) = y^{-2} \exp(-\frac{a_{\vec{\Upsilon}} y + b_{\vec{\Upsilon}}}{y^2})}$$

The fixed point problem has a one parameter family of solutions, given by

$$\Upsilon_2 = (1 + \frac{1}{\lambda})\Upsilon_1^2, \ \forall \Upsilon_1$$

and the corresponding local equilibrium is given by

$$\boxed{f_{equ}(x,y) = \frac{\rho}{Z_{\Upsilon_1}} \frac{1}{y^{\lambda+3}} \exp\left(-\frac{(1+\lambda)\Upsilon_1}{y}\right)}$$

i.e. again by an inverse $\Gamma-$ distribution, giving the 'fat Pareto tails'.

MACROSCOPIC BALANCE LAWS

- The difference to the binary interaction model is that the operator $C(f, \phi_{\vec{\Upsilon}}) = \partial_y [f \partial_y \phi_{\vec{\Upsilon}} + \omega \partial_y (y^2 f)]$ does not conserve y.
- **So we have two parameters $\rho(x,t), \Upsilon_1(x,t)$ in the local equilibrium, but only one conservation law.**

GENERALIZED COLLISION INVARIANTS (GCI'S)[36]

- **Idea:** (Degond, Motsch; 2009) Find $C_f(y)$ such that $\int C_f(y)Q(f)\,dy = 0$ holds on a manifold containing the local equilibrium $f_{loc}(x,y,t)$!
- Q does not conserve f for all solutions, but the moment vanishes in the local equilibrium.
- Gives a (non- conservative) large time equation in the hydrodynamic limit of the form

$$\int C_{f_{loc}} \partial_t f_{loc}\, dy + \int C_{f_{loc}} \nabla_x \cdot [V(x,y) f_{loc}]\, dy = 0$$

- **This equation evolves on the macroscopic time scale, but is not conservative, since $C_{f_{loc}}$ depends on the spatial variable x and time.**

In the case $\phi_{\vec{\Upsilon}} = \frac{a_{\vec{\Upsilon}}}{2}(y - b_{\vec{\Upsilon}})^2$ with $a_{\vec{\Upsilon}} = \frac{\Upsilon_2}{\Upsilon_2 - \Upsilon_1^2}$ and $b_{\vec{\Upsilon}} = (1+\lambda)\Upsilon_1$, the GCI is given by

$$C_{f_{loc}} = C_{\Upsilon_1}(x,y,t) = y\left(\frac{y}{2} - \Upsilon_1(x,t)\right)$$

HYDRODYNAMIC EVOLUTION EQUATIONS FOR THE NON-CONSERVATIVE ECONOMY

The macroscopic system for the local agent density ρ and mean wealth Υ_1 is of the form

$$\boxed{\partial_t \rho + \nabla_x \big(\rho u_0\big) = 0}$$

$$\boxed{\rho \partial_t \Upsilon_1 + \frac{\lambda}{2\Upsilon_1} \nabla_x \cdot (\rho u_2) - \lambda \nabla_x \cdot (\rho u_1) - \frac{1-\lambda}{2} \Upsilon_1 \nabla_x \cdot (\rho u_0) = 0}$$

with

- $u_k = u_k(x; \Upsilon_1) = \int V(x,y) y^k g_{\Upsilon_1}(y)\, dy,\ k = 0:2$
- $g_{\Upsilon_1(x,t)}(y)$ the Gibbs measure given by the inverse $\Gamma-$ distribution.
- The local Nash equilibrium is given by $\boxed{\int y^2 g_{\Upsilon_1}\, dy = (1 + \frac{1}{\lambda}) \Upsilon_1^2}$

Summary 40

- Interplay between Kinetic Theory and Game Theory
  - Best-reply strategy
  - Nash equilibria are Kinetic equilibria of associated dynamics
- Used this analogy to derive:
  - large-scale evolution of system of agents subject to fast relaxation towards Nash equilibrium
  - Hydrodynamic models of games
- Application to wealth distribution
  - Equilibria are inverse gamma distributions
  - Parameters evolve through system of macroscopic equations
  - Applied to non-conservative economy through GCI concept

Perspectives:

- Development in other contexts of social dynamics
- Comparisons with data in real-world applications
- Rigorous proofs