Hyperbolic Quadrature Method of Moments

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The quadrature method of moments (QMOM) reconstructs a velocity distribution function (VDF) from its integer moments: $\{M_0, M_1, \ldots, M_{2N-1}\}$. The reconstructed VDF is a sum of weighted Dirac delta functions in phase space, and closes the spatial flux (M_{2N}) in the kinetic equation. The QMOM closure for M_{2N} leads to a *weakly hyperbolic* system of moment equations. Here, we present an alternative closure where the moment M_5 is a function of $\{M_0, M_1, \ldots, M_4\}$ chosen such that the five-moment system is hyperbolic. We refer to the VDF reconstruction with this choice for M_5 as the *hyperbolic quadrature method of moments* (HyQMOM) reconstruction.

For HyQMOM, we show that (1) a choice for M_5 exists that is valid for realizable moments $\{M_0, M_1, M_2, M_3, M_4\}$, (2) the five eigenvalues of the moment system can be computed explicitly, and (3) the kinetic-based (KB) flux for the system depends on four of the five eigenvalues. In the limit where M_4 is on the boundary of moment space, the KB flux reduces to the 2-node QMOM flux, while for Gaussian moments it corresponds to a 4-node Gauss–Hermite quadrature. A 1-D Riemann problem is solved with HyQMOM to illustrate its ability to handle non-equilibrium VDF without creating delta shocks.

For a multi-variate VDF, a hyperbolic modification of the conditional quadrature method of moments (CHyQMOM) has been developed. For example, in 2-D phase space bivariate moments (i.e. $M_{i,j} : 0 \le i + j \le 3, (i, j) \in (4, 0), (0, 4)$) can be controlled thanks to a judicious choice of the nine velocity abscissas. CHyQMOM reconstructions for moments $M_{i,j,k}$ employ 27 velocity abscissas. The KB fluxes in 2/3-D are defined using the 1-D eigenvalues and directional splitting. Results for 2-D and 3-D crossing jets flows solved with CHyQMOM are presented to demonstrate its ability to capture binary crossing without dispersion.