Robust guiding and control of light and sound in photonic and acoustic metamaterials

Alexander B. Khanikaev

1Electrical Engineering and Physics, City College of New York, 2Physics Department, Graduate Center of CUNY,

Collaborators: Andrea Alu, UT Austin Yuri Kivshar, ANU
Topological roadmap: From Quantum Hall effect to Topological insulators

Broken TR symmetry

\[ H = H_0 - \mu_B S \cdot B \]

One-way edge states

\[ H = H_0 - \chi_{SO} S \cdot L \]

Spin-locked one-way edge states

All the advantages of topological protection without magnetic bias!

Robust edge states in the gap


From condensed matter to photonics: photonic crystals – semiconductors of light

Topological order for photons

Broken TR symmetry and Floquet

Nature Photon. 6, 782–787 (2012)

Preserved TR symmetry


Broken TR symmetry and Floquet


Preserved TR symmetry


Analogue of Quantum Hall Effect:
One-Way Edge States in 2D Magnetic PC

Topological order for photons

\[ \left[ \nabla + i \mathbf{A}(\mathbf{r}) \right]^2 + \tilde{V}(\mathbf{r}) \psi(\mathbf{r}) = 0 \]

\[ \mathbf{A} = \tilde{\mu}/2 [\nabla \times \mathbf{\eta}(\mathbf{r})] \hat{z} \]

\[ \mathcal{A}(\mathbf{k}) = \langle \psi(\mathbf{r}) | \nabla k | \psi(\mathbf{r}) \rangle \]

\[ C = \frac{1}{2\pi i} \int_{\text{BZ}} d^2k [\nabla_k \times \mathcal{A}]_z \]
Topological order for photons

\( -[\nabla + i\vec{A}(\mathbf{r})]^2 + \tilde{V}(\mathbf{r})] \psi(\mathbf{r}) = 0 \)

\( \vec{A} = \vec{\mu}/2[\nabla \times \vec{\eta}(\mathbf{r})]\hat{z} \)

\( \mathcal{A} (\mathbf{k}) = \langle \psi(\mathbf{r})|\nabla_{\mathbf{k}}|\psi(\mathbf{r}) \rangle \)

\[ C = \frac{1}{2\pi i} \int_{\text{BZ}} d^2 k [\nabla_{\mathbf{k}} \times \mathcal{A}]_z \]

\( \{ -[\nabla \pm i\vec{A}(\mathbf{r})]^2 + \tilde{V}^{\uparrow(\downarrow)}(\mathbf{r})\} \psi^{\uparrow(\downarrow)}(\mathbf{r}) = 0 \)

\( \vec{A} = \vec{\mu}/2[\nabla \times \vec{\zeta}(\mathbf{r})]\hat{z} \)

\( \mathcal{A}^{\uparrow(\downarrow)} (\mathbf{k}) = \langle \psi^{\uparrow(\downarrow)}(\mathbf{r})|\nabla_{\mathbf{k}}|\psi^{\uparrow(\downarrow)}(\mathbf{r}) \rangle \)

\[ C^{\uparrow(\downarrow)} = \frac{1}{2\pi i} \int_{\text{BZ}} d^2 k [\nabla_{\mathbf{k}} \times \mathcal{A}^{\uparrow(\downarrow)}]_z \]
**Topological order for photons**

<table>
<thead>
<tr>
<th>Topologically trivial photonic lattice</th>
<th>TR broken topological photonic lattice</th>
<th>TR invariant topological photonic lattice</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram of topologically trivial photonic lattice" /></td>
<td><img src="image2" alt="Diagram of TR broken topological photonic lattice" /></td>
<td><img src="image3" alt="Diagram of TR invariant topological photonic lattice" /></td>
</tr>
<tr>
<td>Nontopological surface state</td>
<td>Chiral edge states</td>
<td>Helical edge states</td>
</tr>
<tr>
<td>$\delta k_{</td>
<td></td>
<td>}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Backscattering $C_1 = 0$</th>
<th>Robust one-way propagation $C_2 = 1$</th>
<th>Robust two-way propagation $C_{2^{\uparrow\downarrow}} = \pm 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_2 = 0$</td>
<td>$C_1 = 0$</td>
<td>$C_{2^{\uparrow\downarrow}} = 0$</td>
</tr>
</tbody>
</table>
Role of Symmetry and Gauge Potentials in Topological Phases

Preserved TR symmetry ensures the presence of Kramer’s TR partners (two spins/helicities) in fermionic systems but not in bosonic.

**Fermions**
\[ \hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V}_{SO} \]

- \( \hat{\mathcal{H}}_0 \) - unperturbed fermionic lattice potential
- \( \hat{V}_{SO} = -\chi_{SO} \hat{\mathbf{S}} \cdot \hat{\mathbf{L}} \) - gauge (SO) potential inducing topological transition (band crossing)

\[ \hat{f}_f \hat{\mathcal{H}} \hat{f}_f^{-1} = \hat{\mathcal{H}} \text{ and } \hat{f}_f^2 = -1 \]

Robustness is insured by TR symmetry (no magnetic defects are allowed).
Doublets generated by TR are locked to their propagation directions – spin-locking.

**Classical/Bosons**
\[ \hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V}_{gauge} \]

- \( \hat{\mathcal{H}}_0 \) - unperturbed bosonic lattice potential
- \( \hat{V}_{gauge} = \text{Photonic} \) gauge potential (SO or pseudo-magnetic) inducing topological transition

\[ \hat{b}_b \hat{\mathcal{H}} \hat{b}_b^{-1} = \hat{\mathcal{H}} \text{ and } \hat{b}_b^2 = 1 \]

**Consequence:** TR alone is not sufficient for topological order for bosons, i.e. no topological phase analogous to fermionic TR phase is possible.
**Solution:** non-TR symmetry protected phases.

Where \( \hat{b}_b \) is a spatial or internal symmetry operator generating a doublet state – pseudo-spin degree of freedom.

\[ \hat{b}_b \psi_\uparrow(\downarrow)(\mathbf{k}) = \hat{b}_b \psi_\downarrow(\uparrow)(-\mathbf{k}) \]

---

Hasan, M. Z. & Kane, C. L., Rev. Mod. Phys. 82, 3045-3067 (2010).
Role of Symmetry and Gauge Potentials in Topological Phases

**Photonic topological insulator:**

I. Duality of EM field as the pseudo-spin generating symmetry

Duality in free space follows by the symmetry of Maxwell equations with respect to electric and magnetic fields:

\[ \hat{D}(E, H) \rightarrow (-H, E) \]

Broken by materials response \( \hat{\epsilon} \neq \hat{\mu} \), it can be restored by (meta-)material’s design.

In dual material \( \epsilon_{zz} = \mu_{zz} \), \( \epsilon_{\perp} = \mu_{\perp} \), duality transformation operator, which satisfies \( \hat{D}^2 = -1 \), allows emulating spin degree of freedom.

\[ \psi^\pm(r; k) = E_z(r; k) \pm H_z(r; k) \]

\[ \hat{F}_b \psi^\pm(r; k) = \psi^\top(r; -k) \]
Role of Symmetry and Gauge Potentials in Topological Phases

**Photonic topological insulator:**
II. Bianisotropy as the gauge field

\[ D = \hat{\epsilon}E + i\hat{\chi}H \quad \text{and} \quad B = \hat{\mu}H - i\hat{\chi}^T E, \]
where \( \hat{\chi} = \begin{pmatrix} 0 & \chi & 0 \\ -\chi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \)

\[ \hat{\mathcal{H}} = v_D\hat{\xi}_0\hat{s}_0\hat{\sigma}_\parallel \cdot \delta k_\parallel + m\hat{\xi}_3\hat{s}_3\hat{\sigma}_3 \]

\( l = \pm 1 \) (dipolar) degeneracy

Topological Order in Metamaterials

Known approach: gyroelectric \( \mathbf{D} = \hat{\epsilon}\mathbf{E} \) or gyromagnetic response \( \mathbf{B} = \hat{\mu}\mathbf{H} \)

\[
\begin{pmatrix}
0 & \nabla \times \\
\nabla \times & 0
\end{pmatrix}
\begin{pmatrix}
\mathbf{E} \\
\mathbf{H}
\end{pmatrix}
= \frac{i\omega}{c}
\begin{pmatrix}
\hat{\epsilon} & 0 \\
0 & \hat{\mu}
\end{pmatrix}
\begin{pmatrix}
\mathbf{E} \\
\mathbf{H}
\end{pmatrix}

Our approach: bianisotropy or magneto-electric coupling*

More general constitutive relations \( \mathbf{D} = \hat{\epsilon}\mathbf{E} + (\hat{\chi} - i\hat{\kappa})\mathbf{H} \) and \( \mathbf{B} = \hat{\mu}\mathbf{H} + (\hat{\chi}^T + i\hat{\kappa})\mathbf{E} \)

\[
\begin{pmatrix}
0 & \nabla \times \\
\nabla \times & 0
\end{pmatrix}
\begin{pmatrix}
\mathbf{E} \\
\mathbf{H}
\end{pmatrix}
= \frac{i\omega}{c}
\begin{pmatrix}
\hat{\epsilon} & \hat{\chi} - i\hat{\kappa} \\
\hat{\chi}^T + i\hat{\kappa} & \hat{\mu}
\end{pmatrix}
\begin{pmatrix}
\mathbf{E} \\
\mathbf{H}
\end{pmatrix}

Bi-isotropic materials
(chiral molecules, subwavelength helixes)

Bi-anisotropic materials/metamaterials (split-rings and \(\Omega\)-particles)

*\( \hat{\kappa} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix} \)

*\( \hat{\kappa} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \)

QSHE as two copies of QHE in electromagnetic systems

Pseudo-gyroelectric + pseudo-gyromagnetic

More general constitutive relations $D = \hat{\epsilon}E + i\hat{\chi}H$ and $B = \hat{\mu}H - i\hat{\chi}^T E$

$$
\begin{pmatrix}
0 & \nabla \times \\
-\nabla \times & 0
\end{pmatrix}
\begin{pmatrix}
E \\
H
\end{pmatrix}
= \frac{i\omega}{c}
\begin{pmatrix}
\hat{\epsilon} & i\hat{\chi} \\
-i\hat{\chi}^T & \hat{\mu}
\end{pmatrix}
\begin{pmatrix}
E \\
H
\end{pmatrix}
$$

Bianisotropic metamaterials (split-rings and $\Omega$-particles)

$$
\hat{\chi} = \begin{pmatrix}
0 & \chi & 0 \\
-\chi & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\quad \quad \quad
\begin{pmatrix}
0 & \nabla \times \\
-\nabla \times & 0
\end{pmatrix}
\begin{pmatrix}
E \\
H
\end{pmatrix}
= \frac{i\omega}{c}
\begin{pmatrix}
\hat{\epsilon} & i\hat{\chi} \\
-i\hat{\chi}^T & \hat{\mu}
\end{pmatrix}
\begin{pmatrix}
E \\
H
\end{pmatrix}
$$

If $\hat{\epsilon} = \hat{\mu}$ after simple transformation $\psi^+ = E + H$ and $\psi^- = E - H$

$$
\begin{pmatrix}
0 & \nabla \times \\
-\nabla \times & 0
\end{pmatrix}
\begin{pmatrix}
\psi^+ \\
\psi^-
\end{pmatrix}
= i\omega
\begin{pmatrix}
\hat{\epsilon} + i\hat{\chi} & 0 \\
0 & \hat{\mu} - i\hat{\chi}
\end{pmatrix}
\begin{pmatrix}
\psi^+ \\
\psi^-
\end{pmatrix}
$$

Which are exact two copies of electromagnetic QHE for $\psi^+$ and $\psi^-$ with inverted effective magnetic fields

$$
\psi^+: \hat{\epsilon} + i\hat{\chi} =\begin{pmatrix}
\epsilon_{xx} & i\chi & 0 \\
-i\chi & \epsilon_{yy} & 0 \\
0 & 0 & \epsilon_{zz}
\end{pmatrix} \\
\psi^-: \hat{\epsilon} - i\hat{\chi} =\begin{pmatrix}
\epsilon_{xx} & -i\chi & 0 \\
i\chi & \epsilon_{yy} & 0 \\
0 & 0 & \epsilon_{zz}
\end{pmatrix}
$$
Quantum Spin Hall Effect in Metamaterials

2-D Meta-Crystal: Photonic Crystal comprised of metamaterial inclusions

\[
\begin{align*}
(k_0^2 \mu_{zz} + \nabla_\perp \frac{1}{\epsilon_\perp} \nabla_\perp) H_z &= \left[ \nabla_\perp \left( \frac{-i \chi_{xy}}{\epsilon_\perp \mu_\perp} \right) \times \nabla_\perp E_z \right]_z, \\
(k_0^2 \epsilon_{zz} + \nabla_\perp \frac{1}{\mu_\perp} \nabla_\perp) E_z &= \left[ \nabla_\perp \left( \frac{-i \chi_{xy}}{\epsilon_\perp \mu_\perp} \right) \times \nabla_\perp H_z \right]_z,
\end{align*}
\]

\[
(k_0^2 \epsilon_{zz} + \nabla_\perp \frac{1}{\mu_\perp} \nabla_\perp) \psi^\pm = \pm \left[ \nabla_\perp \left( \frac{-i \chi_{xy}}{\epsilon_\perp \mu_\perp} \right) \times \nabla_\perp \psi^\pm \right]_z
\]

\[
\psi^\pm(x_\perp; q) = E_z(x_\perp; q) \pm H_z(x_\perp; q)
\]

- By restoring polarization degeneracy $\epsilon_{zz} = \mu_{zz}$, $\epsilon_\perp = \mu_\perp$ we emulate spin degree of freedom
- Biaxiality works as an effective spin-orbital coupling responsible for the topological transition

\[
T \psi^\pm(x_\perp; q) = \psi^\mp(x_\perp; -q)
\]

Photonic Topological Insulators: QSHE

Nothing new... unless the bianisotropy is **ON**

- Without bianisotropy (no gap): photonic crystal with double Dirac point
- With bi-anisotropy (gapped PBS): photonic QSHE insulator

\[ l = \pm 1 \text{ (dipolar) degeneracy} \]
Topological invariants of the photonic topological insulator

\[ A_n^{\pm} = -i \langle \psi_n^{\pm}(k) | \nabla_k | \psi_n^{\pm}(k) \rangle \]

\[ C_n^{\pm} = \pm \frac{1}{2\pi} \int_{BZ} d^2k \left[ \partial_{k_x} A_y^{n\pm}(k) - \partial_{k_y} A_x^{n\pm}(k) \right] \]

\[ \psi^+ = E_z + H_z \]

\[ \psi^- = E_z - H_z \]

\[ C_1^+ = 1 \]

\[ C_1^- = -1 \]

\[ C_2^+ = -1 \]

\[ C_2^- = 1 \]

Matching gaps of trivial and nontrivial insulators to avoid leaks

Selective excitation of spin-up and spin-down edge states
Topological protection of spin-locked edge states

(i) Robustness against scattering by sharp bends

(ii) Tunneling through the cavity at any frequency within the gap (not only at Fabry-Perot resonances)

Practical designs of photonic topological insulators


Effective Kane-Mele-like Hamiltonian
\[ \hat{H}_{\uparrow/\downarrow} = v_D \hat{t}_0 \hat{s}_0 \hat{\sigma}_\parallel \cdot \delta \hat{k}_\parallel + m \hat{t}_3 \hat{s}_3 \hat{\sigma}_3 \]

Non-vanishing spin-Chern numbers
\[ C_{u/l}^{\uparrow} = \pm \frac{m}{|m|} \text{ and } C_{u/l}^{\downarrow} = \mp \frac{m}{|m|} \]

Topological edge states

Reconfigurable domain wall

\[ m > 0 \quad \text{Domain wall} \quad m < 0 \]

Static non-reconfigurable interface

\[ m > 0 \quad \text{Domain wall} \quad m = 0 \]

Reconfigurable guiding along arbitrarily shaped pathways

Two 60 deg. bends

Two 90 deg. bends

Two 120 deg. bends

Exponential localization of the edge states

--- Transmission through the bulk

--- Transmission through the domain wall

Robustness against disorder

Experimental demonstration of ballistic transport of the topological edge modes through randomly shaped domain walls and disordered regions

Demonstration of spin-locking of the topological edge states

Experimental proof of spin-locked wave-division of an edge mode at a four-port topological junction.

Motivation to move to “all-dielectric” and 3D

i) Moving to optical domain
ii) Rich 3D physics: Weyl points and “true” Dirac points
iii) Avoiding magnetic materials: symmetry-protected topological order without breaking TR


2D all-dielectric photonic topological metasurface

Degeneracy between magnetic and electric dipolar modes of the cylinders + all-dielectric bianisotropy

Even (+):
- +
- +

Odd (-):
- +
- -

Electric dipolar
Magnetic dipolar

Double Dirac cone
Reversal of bianisotropy to create topological domain walls

\[ \hat{H}_{\uparrow/\downarrow} = v_D \hat{\mathbf{t}}_0 \hat{s}_0 \mathbf{\hat{\sigma}}_\parallel \cdot \mathbf{\delta k}_\parallel + m \hat{\mathbf{t}}_3 \hat{s}_3 \mathbf{\hat{\sigma}}_3 \]

\( m > 0 \) \quad \rightarrow \quad \hat{H}_{\uparrow/\downarrow} \quad \rightarrow \quad m < 0 \)
Experimental realization
Experiment: spin-locking
Experiment: sharp bends
Weak topological insulator – stacking of 2D TIs

Degeneracy between magnetic and electric dipolar modes of the cylinders + all-dielectric bianisotropy

\[ \hat{H} = \omega_0 + v_\parallel \hat{s}_0 (\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y) + v_z \hat{s}_y \hat{\sigma}_x \delta k_z + m \hat{s}_z \hat{\sigma}_z \]

Electromagnetic perturbation theory

\[ \hat{\mathcal{H}}_K = \omega_0 \hat{S}_0 \hat{\sigma}_0 \]

**Two mechanisms of bianisotropy:**

1. Even (+):
   - \[ + \]

2. Odd (-):
   - \[ +, - \]

**Electric (E) & Magnetic (M) Perturbation:**

\[ \Delta_{mn} = -\int d^3r \, \delta \varepsilon_r \varepsilon_0 \mathbf{E}_n \cdot \mathbf{E}_m^* \]

**Dielectric perturbation**

\[ \{S_i\}_{mn} = \int d^3r \left\{ \mathbf{E}_n^* \times \mathbf{H}_m + \mathbf{E}_m \times \mathbf{H}_n^* \right\}_{i} \]

**3D Dirac Hamiltonian**

\[ \hat{\mathcal{H}} = \omega_0 \hat{S}_0 \hat{\sigma}_0 + v_{||} \hat{S}_0 \left( \delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y \right) + v_z \hat{S}_y \hat{\sigma}_z \delta k_z + m \hat{S}_z \hat{\sigma}_z \]

3D Dirac Hamiltonian!
Topological edge states of 2D domain walls

\[ \hat{\mathcal{H}} = \omega_0 + v || \hat{s}_0 (\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y) + v_z \hat{s}_y \hat{\sigma}_z \delta k_z \pm m \hat{s}_z \hat{\sigma}_z \]

Jackiw-Rebbi-like surface states:

\[ \Omega_\pm = \pm \sqrt{\zeta^2 + v_F^2 (k_x^2 + k_y^2)} + v_z^2 k_z^2 \]
Spin-locking of edge states on 2D domain walls

\[ \hat{H} = \omega_0 + v_{||}\hat{s}_0(\delta k_x\hat{\sigma}_x + \delta k_y\hat{\sigma}_y) + v_z\hat{s}_y\hat{\sigma}_z\delta k_z \pm m\hat{s}_z\hat{\sigma}_z \]

Jackiw-Rebbi-like surface states:

\[ \Omega_{\pm} = \pm \sqrt{\zeta^2 + v_F^2(k_x^2 + k_y^2) + v_z^2k_z^2} \]

\[ \psi^{(1)}_{s_{\pm}} \sim iv_{\perp}k_z|e_1 + h_1\rangle + (v_{||}k_y - \Omega_{s_{\pm}})|e_1 - h_1\rangle \]

\[ \psi^{(1)}_{s_{\pm}}^{\dagger}\hat{s}\psi^{(1)}_{s_{\pm}} = \frac{v_{\perp}k_z}{\Omega_{s_{\pm}}}\hat{y} + \frac{v_{||}k_y}{\Omega_{s_{\pm}}}\hat{z} \]
Topological robustness in three-dimensions

Reflectionless routing around sharp corners for out-of-plane ($k_z \neq 0$) propagation
Non-topological surface states with Dirac point insured by the hexagonal symmetry of the defect
Acoustic and elastic topological states

1) Acoustic analogue of Quantum Hall effect

In collaboration with Andrea Alu (UT Austin)

2) Acoustic analogue of Quantum Spin Hall Effect

In collaboration with Hossein Mousavi and Zheng Wang (UT Austin)

3) Floquet Topological Insulators for Sound
   Nature Communications 7, 11744 (2016).
Emulating spin-orbit coupling and transition to phononic QSHE

Opening a counterbore breaks $\sigma_z$, while preserves in-plane symmetries.

Berry curvature near K point:

Bottom bands

...and couple S and A modes

5% complete topological bandgap

Bottom bands

Top bands
Massless helical edge states, spin locked to the propagation direction.

Time reversal operation changes the direction as well as the spin.
Trivial crystals are prone to defects and disorders. Each time a resonance occurs, phase changes by $\pi$ and transmission drops to zero.
Robustness against defects and disorders in Quantum Spin Hall Effect crystal

Resonance manifests itself only in the phase!

FEM simulations
Robustness against sharp bends and rerouting in Quantum Spin Hall Effect crystal
Summary and Outlook

• Photonic and acoustic systems offer an ideal platform for emulating topological states of condensed matter and quantum relativistic systems.
• Topological edge states envision a broad range of applications such as reconfigurable waveguides with controllable routing along the domain walls, and integrated optical systems where interaction among optical elements has “one-way” character.
Thank you!

Research sponsored by National Science Foundation (USA)

CMMI-1537294 and EFRI-1641069