Hypocoercivity and Sensitivity Analysis in Kinetic Equations and Uncertainty Quantification

October 2nd – 5th

Department of Mathematics, University of Wisconsin Madison

Venue: van Vleck Hall 911

Monday, October 2	
8:30 – 9:15	**Registration**
9:15 – 9:30	**Opening remarks**
9:30 – 10:20	A. Arnold *Large-time behavior in hypocoercive BGK-models*
10:20 – 10:50	**Coffee**
10:50 – 11:40	J. Dolbeault *Hypocoercivity without confinement I: a mode-by-mode analysis*
12:00 – 14:00	**Lunch**
14:00 – 14:50	C. Schmeiser *Hypocoercivity without confinement II: decay rates in the Euclidean space*
14:50 – 15:40	R. Duan *The Boltzmann equation in Besov spaces with spatially critical regularity*
15:40 – 16:10	**Coffee**
16:10 – 17:00	R. Shu *A stochastic asymptotic-preserving scheme for a kinetic-fluid model for disperse two-phase flows with uncertainty*

Tuesday, October 3	
9:30 – 10:20	Q. Li *TBD*
10:20 – 10:50	**Coffee**
10:50 – 11:40	*L. Liu* *Hypocoercivity and exponential decay to equilibrium and uniform spectral accuracy of the stochastic Galerkin method for collisional kinetic equations with uncertainties and multiple scales*
12:00 – 14:00	**Lunch**
14:00 – 14:50	J.-G. Liu *TBD*
14:50 – 15:40	Y. Zhu *TBD*
15:40 – 16:10	**Coffee**
16:10 – 17:00	H. Lu *An Asymptotic-preserving stochastic Galerkin method for the radiative heat transfer equations with random inputs and diffusive scalings*
18:00 – 20:00	**Conference dinner**

Wednesday, October 4	
9:30 – 10:20	W. Jing *TBA*
10:20 – 10:50	**Coffee**
10:50 – 11:40	R. Strain *TBD*
12:00 – 14:00	**Lunch**
14:00 – 14:50	I. Gamba *Boundary data for the Boltzmann Poisson system with rough boundaries*
14:50 – 15:40	L. Pareshi *Multiscale Monte Carlo methods for uncertainty quantification of kinetic equations*
15:40 – 16:10	**Coffee**

16:10 – 17:00	M.-B. Tran *Sensitivity analysis for hyperbolic conservation laws*

Thursday, October 5	
9:30 – 10:20	F. Golse *TBA*
10:20 – 10:50	**Coffee**
10:50 – 11:40	L. Wang *Uniform regularity for linear kinetic equations with random input*
11:40 – 12:00	**Closing Remarks**

List of Abstracts

Anton Arnold: BGK equations are kinetic transport equations with a relaxation operator that drives the phase space distribution towards the spatially local equilibrium, a Gaussian with the same macroscopic parameters. Due to the absence of dissipation w.r.t. the spatial direction, convergence to the global equilibrium is only possible thanks to the transport term that mixes various positions. Hence, such models are hypocoercive. We shall prove exponential convergence towards the equilibrium with explicit rates for several linear, space periodic BGK-models in dimension 1 and 2. Their BGK-operators differ by the number of conserved macroscopic quantities (like mass, momentum, energy), and hence their hypocoercivity index. Our discussion includes also discrete velocity models, and the local exponential stability of a nonlinear BGK-model. The proof is based, first, on a Fourier decomposition in space and Hermite function decomposition in velocity. Then, the crucial step is to construct a problem adapted Lyapunov functional, by introducing equivalent norms for each mode. References: * F. Achleitner, A. Arnold, E.A. Carlen: On linear hypocoercive BGK models; in Springer Proceedings in Mathematics & Statistics, Vol. 126, 2016; p. 1-37. * F. Achleitner, A. Arnold, E.A. Carlen: On multi-dimensional hypocoercive BGK models; preprint 2017.

Jean Dolbeault: L2 hypocoercivity methods applied to scattering and Fokker-Planck type collision operators will be presented using decoupled Fourier modes. The results are obtained on a space with exponential weights and then extended to larger function spaces by a factorization method. The values of the estimates of the rates will be discussed.

Christian Schmeiser: We prove decay to zero at the same rate as the heat equation, for kinetic models with diffusive macroscopic behavior, meaning algebraic decay in an unconfined setting. A hypercoercivity approach is combined with an application of the Nash inequality, which can also be used to obtain the decay behavior for parabolic equations. As for the heat equation, the decay rate can be improved for massless initial data.

Renjun Duan: We study the long-time behavior of solutions to the Cauchy problem on the nonlinear Boltzmann equation in the whole space whenever initial data is sufficiently close to a global Maxwellian in a velocity-weighted Besov space with spatially critical regularity. Both hard and soft potentials with angular cutoff are considered. Precisely, in case of hard potentials, we establish the global existence of mild solutions basing on the spectral analysis and the fixed point argument, and also obtain the polynomial rate of convergence provided that the initial perturbation additionally belongs to some Lebesgue space. The interesting point occurs in the case of very soft potentials where the result of the semigroup theory is not available, and we are still able to obtain the global-in-time mild solution by carrying out an interpolation technique combined with the time-decay properties for the linearized equation through the purely energy method. This is a recent work joint with Shota Sakamoto in Kyoto University.

Ruiwen Shu: We consider a kinetic-fluid model for disperse two-phase flows with uncertainty. We propose a stochastic asymptotic-preserving (s-AP) scheme in the generalized polynomial chaos stochastic Galerkin (gPC-sG) framework, which allows the efficient computation of the problem in both kinetic and hydrodynamic regimes. The s-AP property is proved by deriving the equilibrium of the gPC version of the Fokker-Planck operator. The coefficient matrices that arise in a Helmholtz equation and a Poisson equation, essential ingredients of the algorithms, are proved to be positive definite under reasonable and mild assumptions. The computation of the gPC version of a translation operator that arises in the inversion of the Fokker-Planck operator is accelerated by a spectrally accurate splitting method. In the light particle regime, using energy estimates and hypocoercivity arguments, we prove the uniform regularity in the random space of the model for random initial data near the global equilibrium in some suitable Sobolev spaces. Then we prove that the gPC-sG method has spectral accuracy, uniformly in time and the Knudsen number. Numerical examples illustrate the s-AP property and the efficiency of the gPC-sG method in various asymptotic regimes.

Qin Li: TBA

Liu Liu: The speaker first reviews the hypocoercivity assumptions for general deterministic kinetic models, including the Boltzmann, Landau and semi-classical quantum relaxation equations etc, based on the framework constructed by Mouhot-Neumann (06') and Marc Briant (15'). The multiple scalings considered include the compressible Euler and the incompressible Navier-Stokes, in particular for the Boltzmann equation. Uncertainties, arisen from measurement errors etc, can enter into the system through both the initial data and collision kernels. It is crucial to study how uncertainties affect the solution as time goes to infinity. An exponential decay of the solution to the global equilibrium is established under some norm that is equivalent to the standard Sobolev norms in the physical space. We will also show the gPC stochastic Galerkin method has a spectral accuracy, under the assumption that the initial data is smooth enough and close to the global equilibrium. The gPC approximation error also decays exponentially in time. This is a joint work with Prof. Shi Jin.

Jian-Guo Liu: TBD

Yuhua Zhu: TBD

Hanqing Lu: In this poster, we present an Asymptotic-Preserving (AP) stochastic Galerkin scheme for the radiative heat transfer equations with random inputs and diffusive scalings. In this problem the random inputs arise due to uncertainties in cross section, initial data or boundary data. We use the generalized polynomial chaos based stochastic Galerkin (gPC-SG) method, which is combined with the even-odd decomposition based deterministic AP framework in order to handle efficiently the diffusive regime. To improve the CFL condition, we used a IMEX-RK method. It is remarkably efficient especially for the UQ problems. For linearized problem we prove the regularity of the solution in the random space and consequently the spectral accuracy of the gPC-SG method. We also prove the uniform (in the mean free path) linear stability for the space-time discretizations. Several numerical tests are presented to show the efficiency and accuracy of proposed scheme, especially in the diffusive regime.

Eric Carlen: TBD

Robert Strain: TBD

Irene Gamba: We shall discuss a set up for the Boltzmann - Poisson system with rough boundary conditions corresponding to zero flux. I'll present a a detailed numerical study of specular vs diffusive reflection conditions for a two dimensional domain with transport in full phase k-space. We'll discuss some estimates as well as issues with the numerical implementation. This is work in collaboration with Jose Morales Escalante (TU Wien).

Lorenzo Pareschi: In this talk we will present some recent results on the construction of efficient Monte Carlo methods for uncertainty quantification in kinetic equations. The method is based on a suitable micro-macro decomposition strategy combined with the use of different sampling scales on the random field in order to reduce the variance of standard Monte Carlo sampling methods. Applications to socio-economy and rarefied gas dynamics are presented.

Minh Binh Tran: We develop a local sensitivity analysis for the Cauchy problem of viscous conservation laws and systems of conservation laws. The analysis sheds lights on the long time dependence of solutions with respect to random initial input. The talk is based on my joint works with Prof Shi Jin and Prof Enrique Zuazua.

François Golse: TBD

Li Wang: In this talk we consider the effect of randomness in kinetic equations that preserve mass. The analysis is carried out in a general setting, with the regularity result not depending on the specific form of the collision term, the probability distribution of the random variables, or the regime the system is in. The proof relies on the explicit expression of the high order derivatives of the solution in the random space, and the convergence in time is mainly based on hypocoercivity, which, despite the popularity in PDE analysis of kinetic theory, has rarely been used for numerical algorithms. This is joint work with Qin Li.